International Journal of Non-Linear Mechanics 46 (2011) 942–948

Contents lists available at ScienceDirect

International Journal of Non-Linear Mechanics

journal homepage: www.elsevier.com/locate/nlm

The boundary layers of an unsteady incompressible stagnation-point flow with mass transfer

Tiegang Fang [a,*], Chia-fon F. Lee [b,c], Ji Zhang [a]

[a] Department of Mechanical and Aerospace Engineering, North Carolina State University, NC 27695, USA
[b] Center for Combustion Energy and State Key Laboratory of Automotive Safety and Energy, Tsinghua University, China
[c] Department of Mechanical Science and Engineering, University of Illinois at Urbana-Champaign, Urbana, IL 61801, USA

ARTICLE INFO

Article history:
Received 1 August 2008
Accepted 5 April 2011
Available online 15 April 2011

Keywords:
Similarity solution
Mass transfer
Unsteady boundary layer
Stagnation-point flow

ABSTRACT

In the current work, the boundary layers of an unsteady incompressible stagnation-point flow with mass transfer were further investigated. Similarity transformation technique was used and the similarity equation group was solved using numerical methods. Interesting observation is that there are multiple solutions seen for negative unsteadiness parameters, β. The influences of mass transfer, unsteadiness parameter, and Prandtl numbers on velocity and temperature profiles, wall drag, and wall heat fluxes were investigated and analyzed. The asymptotic behaviors for the similarity equations in limiting situations were theoretically analyzed. It is found that solutions exist for all mass transfer parameters for $\beta \geq -1$. For a certain mass transfer parameter, there are two solutions when $\beta_c < \beta < 0$; there is one solution for $(\beta = \beta_c) \cup (\beta \geq 0)$; there is no solution for $\beta < \beta_c$, where β_c is a critical unsteadiness parameter dependent on mass transfer parameter.

1. Introduction

Unsteady boundary layer plays important roles in many engineering problems like start-up process and periodic fluid motion. Unsteady boundary layer has different behavior due to extra time-dependent terms, which will influence the fluid motion pattern and the boundary layer separation [1–3]. Some typical examples of unsteady boundary layers in the history of fluid mechanics are the Rayleigh problem and Stokes oscillating plate [2,3]. The governing equation of two-dimensional unsteady boundary layers reads [4]

$$\psi_{yyy} + \psi_x \psi_{yy} - \psi_y \psi_{xy} - \psi_{ty} + U_\infty U_{\infty,x} + U_{\infty,t} = 0 \tag{1a}$$

where $\psi(x,y)$ is the stream function, $U_\infty(x,t)$ is the time-dependent free stream velocity, and the subscripts denote differentiation. The solution of Eq. (1) has been studied by many researchers for different $U_\infty(x,t)$ and flow configuration as discussed in the Refs. [3–6] and the references therein. Similarity transformation technique played a significant role in solving the unsteady boundary layer problem by transforming a Partial Differential Equation (PDE) into an Ordinary Differential Equation (ODE) in most studies. Yang [6] investigated the unsteady boundary layer for a stagnation flow involving the starting up of a cylinder. The corresponding free stream velocity was taken as $U_\infty(x,t) = x/(a+bt)$. The thermal boundary layer for a constant temperature wall was also discussed. More interesting similarity solutions were found in Refs. [4,5] by extensive study of the boundary layer Eq. (1). In these unsteady boundary layers, the flow configurations only involve impermeable walls. Following the pioneer work by Yang, the problem was extended to unsteady axis-symmetric stagnation-point flow by Williams III [7] and to a general three dimensional stagnation-point flow by Jankowski and Gersting [8]. The heat transfer behavior for the three dimensional unsteady stagnation-point flow was studied by Teipel [9]. The effect of the unsteadiness parameter was discussed and heat transfer was reduced with increasing unsteadiness parameter. The unsteady oblique stagnation-point flow was investigated by Wang [10]. However, for all these studies there was no mass transfer involved. In another words, the plate was impermeable without no mass suction or mass injection. Mass transfer is important in boundary layer control [2,3] and in optimizing the drag and heat transfer characteristics. An early work, including mass transfer for this unsteady stagnation-point flow was performed by Rajappa [11]. In his paper, the effects of mass blowing on the wall were investigated. Hard blowing on the wall helped reduce the wall shear forces as well as heat transfer rate. But there was no discussion of the effects of mass suction on the wall. Some new solution for the unsteady stagnation-point flow was presented by Burde [12]. Some explicit forms of solutions were given in the paper, which are also exact solutions of the unsteady Navier–Stokes equations. The unsteady stagnation-point boundary layer problem is still a quite active area with many papers published in journals. These works include the axisymmetric stagnation flow over a cylinder [13], mixed

* Corresponding author. Tel.: +919 5155230.
E-mail addresses: tfang2@ncsu.edu (T. Fang), cflee@tsinghua.edu.cn, cflee@illinois.edu (C.F. Lee).

0020-7462/$ - see front matter © 2011 Elsevier Ltd. All rights reserved.
doi:10.1016/j.ijnonlinmec.2011.04.007