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Elastocapillary imbibition

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ABSTRACT

When a wetting liquid invades a porous medium or a capillary tube, the penetration or imbibition speed is known to decrease as the square root of time. We examine the capillary filling of a gap between flexible sheets and demonstrate that the pressure-induced inward deflection of the sheets leads to a non-monotonic behavior of the speed of the invading meniscus until eventually the flow is blocked. A model based on lubrication theory is formulated as a non-linear free-boundary problem, which is solved numerically using finite-difference methods. Good agreement is obtained with our experiments. At early times the deformation of the sheets is insignificant, and the penetration speed is unaffected. At later times, as the penetration distance approaches the elastocapillary length, the deformation becomes appreciable and the flow accelerates. Shortly thereafter, the gap at the air–liquid interface goes to zero, and the flow necessarily stops. The length of the sheets above which imbibition will cause them to coalesce is determined and is found to be in good agreement with that predicted via scaling arguments. Biological applications of this transient wetting of flexible boundaries are discussed.

1. Introduction

The surface-tension-driven coalescence of flexible structures, i.e., elastocapillarity, is relevant to a number of biological and engineering processes, such as the closure of pulmonary airways [1–3], the design of biomimetic adhesives [4,5], the failure of micro and nanoscale devices [6–9], and the stability of water-walking arthropods [10]. Elastocapillarity has been the subject of several recent studies that analyze equilibrium configurations where there is a balance between elasticity and capillarity. Bico et al. [11] and Kim and Mahadevan [12], for example, investigated the statics of capillary rise of a liquid between flexible sheets, and Kwon et al. [13] examined the shape of a liquid drop confined beneath a flexible sheet. To the best of our knowledge, the transient wetting of flexible boundaries has been considered by relatively few authors. Siddique et al. [14] investigated the dynamics of capillary rise into a deformable porous material, while van Honschoten et al. [15] studied the capillary filling, or *imbibition*, of a nanochannel with an elastic capping layer. In this paper we consider imbibition between flexible sheets, and we determine a criterion for their coalescence. In particular, we quantify the time-dependent deflection of the sheets and the penetration speed of the invading liquid as it advances towards the initially free end of the sheets.

Consider the capillary filling of a gap between flexible, inextensible sheets of thickness b, width w (into the page) and length ℓ that are initially parallel and separated by a distance $2h_0$, as depicted in Fig. 1. The sheets are clamped at their left end, where fluid enters from a reservoir at atmospheric pressure, so that the flow into the gap between the sheets is driven solely by surface tension owing to a reduction in pressure at the meniscus; gravitational effects are neglected. The pressure distribution associated with the invasion of fluid leads to an inward deflection of the sheets, which further alters the pressure distribution in the liquid and the speed at which the meniscus advances. The shape of the sheet is denoted $h(z,t)$, and the penetration distance or meniscus position $z_m(t)$.

The characteristic length scale over which the sheets will deflect to a combined distance of $2h_0$ may be estimated by taking each sheet to be a cantilever that is loaded at a distance ℓ_{ec} from its clamped end by the Laplace pressure $\gamma\kappa$, where γ is the surface tension. The curvature at the meniscus may be approximated by $\kappa = \cos\theta_e/h(z_m) + \cos\theta_e/(w/2)$, where θ_e is the contact angle (imbibition requires $\theta_e < \pi/2$) [16]. By taking $h(z_m,t) = h_0$, $h_0 \ll w$, and by using the Euler–Bernoulli beam equation $Bh_{zzzz} = \gamma\kappa$, where subscripts denote derivatives, $B = Eb^3/12(1-\nu^2)$ is the bending stiffness per unit width, E the Young's modulus, and ν the Poisson ratio, we identify the characteristic length scale

$$\ell_{ec} = \left(\frac{Bh_0^2}{\gamma\cos\theta_e}\right)^{1/4}, \qquad (1)$$

which may be interpreted as an elastocapillary length. This particular length scale is also relevant to the coalescence of wet hair [11], and the adhesion-related failure of micromechanical

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