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The bouncing motion of a superball between a horizontal floor and a vertical wall

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ABSTRACT

In earlier work [P.J. Aston, R. Shail, The dynamics of a bouncing superball with spin, Dyn. Sys. 22 (2007) 291–322] the problem of the possible back and forth motion of a superball thrown spinning onto a horizontal plane was considered in detail. In this paper the problem is extended to include a vertical wall. In particular motion of the superball where it bounces alternately on the floor and the wall several times is considered. Using the same physical model as in our previous work, a non-linear mapping is derived which relates the launch data of the $(n+1)$th floor bounce to that of the n th. This mapping is analysed both numerically and theoretically, and a detailed description is presented of various possible motions. Regions of initial conditions which result in a specified number of bounces against the wall are also considered.

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1. Introduction

In a previous paper [1] two of the present authors considered in detail the mechanics of a superball bouncing back and forth on a rough horizontal plane. Reversals in direction in the horizontal motion of the ball result from the application of a tangential law of restitution at the point of impact of the ball and the plane. This concept was first introduced by Garwin [2] who used a tangential coefficient of restitution of one, which is not physically realistic. Garwin's model was modified by Cross [3] who employed a tangential coefficient of restitution α satisfying $0<\alpha<1$, with the horizontal velocity of the point of impact of the ball being reversed and reduced in magnitude by a factor of α in the impact. Further details of the physics of this model are given in [1], together with references to other theoretical and experimental work.

All who have experimented with a superball will have at sometime bounced the ball on the floor, followed by a bounce on a vertical wall. If the bounce on the wall occurs while the ball is still rising, it gives the ball some backspin, so that the direction of motion is reversed at the next bounce on the floor resulting in the ball hitting the wall a second time. With practice, the ball can be made to bounce between the floor and wall several times. Such motion is illustrated in the animations in Figs. 2, 3, 5, and 11. It is our purpose to give a theoretical investigation of such motions and the non-linear mappings which they engender. To this end we establish in Section 2 the basic equations governing the model. Essentially, each journey of the ball from floor to wall to floor, assumed to take place in the same vertical plane, comprises four events: (i) after launch from the floor the ball pursues a parabolic trajectory until it hits the wall, (ii) the rebound from the wall, (iii) the parabolic trajectory of the return journey to the floor and (iv) the impact with the floor which provides the launch data for the next excursion of the ball. The result of this analysis is the derivation of a non-linear mapping which relates the floor launch data (linear and angular velocity components of the ball and distance from the wall) to the same parameters after the next bounce on the floor.

In Section 3 some numerical trajectories of the non-linear mapping are computed and examples given of motions with various numbers of floor to wall bounces. Also illustrated are the parameter spaces of initial conditions required to produce various numbers of bounces off the wall. In Section 4 a scaling invariance is introduced which rewrites the non-linear map of Section 2 in terms of suitable canonical coordinates. This results in a three-dimensional non-linear map, a reduction in dimension by one from the original system.

Section 5 presents some numerical results for the regions of initial conditions which will result in a given number of bounces against the wall in the canonical variables, analogous to those of Section 3 for the original variables. The next two sections of the work analyse these numerical results in some detail, focussing on the behaviour of the mapping on two planes which comprise boundaries of the region of interest. The paper concludes by proposing a number of further questions related to the problem.

Before continuing to our analysis of the problem we have just described, we note that there are limitations to the model of the bounce of the superball that we use. It is recognised that the

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