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The growth of giant pumpkins: How extreme weight influences shape

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ABSTRACT

Great morphological differences exist among fruits and vegetables. In this combined experimental and theoretical study, we predict pumpkin shape evolution and maximum size based on their material properties. Using time-lapse photography and measurements collected by volunteer farmers, we show that as pumpkins grow, they morph from spherical to pancake shapes, flattening up to 50% in height-to-width aspect ratio. By compressing whole pumpkins in material-testing machines, we find that the elastic response of the pumpkin is insufficient to account for the large deformations characteristic of large pumpkins. We hypothesize that pumpkin flattening is caused by the weight of the pumpkin retarding its normal growth processes. We test this hypothesis using a mathematical model that assumes plant growth is stimulated in response to a tensile yield stress. We are able to predict pumpkin shapes consistent with those observed. The observed growth plasticity allows the fruit to redistribute internal stresses, thereby growing to extreme sizes without breaking.

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1. Introduction

The development of shape in plants is a century-old problem [1] that has made recent advances due to the combined interdisciplinary efforts of plant, molecular and mathematical biology. A challenge inherent to this problem is the multiple length scales involved, particularly in embryogenesis or fruit growth, where an ovary of characteristic size $100 \ \mu\text{m}-10 \ \text{mm}$ will reproduce over time to become a 10-cm fruit, or in the case of this investigation, a one-meter fruit. During this vast change in size, the plant genome regulates growth through feedback with the multi-parameter chemical and physical state of the fruit [2]. The chemical and biological changes in cells during growth [3,4] are beyond the scope of this study. Instead, we focus on the use of a simple computational model, which approximates elasto-plastic plant material, to investigate the mechanics of extreme growth.

Given the computational nature of our study, it is worthwhile to briefly review previous mathematical approaches here. Goriely et al. [5] and Taber [6] provide comprehensive reviews of continuum models used to model the growth of plant and animal tissues. A common theme among these models is the decomposition of strain into components due to elasticity and growth. In another study, Vandiver and Goriely [7] explain how differential growth in the plant can generates residual stresses, such as tension, and consequently how these tensions can rigidify and strengthen the plant. Dumais et al. [8] has shown how the behavior of certain materials like rubber balloons can be used as models for root tip growth. Coen et al. [9] review how the growth of blossoms can be computationally modeled using formulations of elasticity and growth rules. While most previous models take one-dimensional or two-dimensional approaches to growth, we take advantage of computational methods that are particularly suited for examining three-dimensional changes.

We apply a lattice spring method (LSM) [10,11], a computational model, to estimate pumpkin deformation. Originally derived for performing atomistic simulations [12], this method can be applied for modeling elastic solids in the continuum mechanics approximation [10,11,13]. Our computational lattice model constitutes a means for examining the influence of the visco-plastic properties [10] on development of fruit shapes under different environmental conditions. Furthermore, by removing individual bonds, LSM allows modeling of crack formation and propagation thorough solid materials [14,15]. Such a scenario is often observed in giant vegetables where cracks may appear as a result of the extremely fast growth.

The study of large organisms can provide us insight into the growth and stress limits of tissues and can provide useful testing grounds for hypotheses about biomechanics. The largest organisms such as trees and dinosaurs, push the envelope of growth, metabolic and respiratory processes occurring in the whole organism and its constituent cells. For instance, the maximum height of redwood trees is 130 m, due to the inability of trees to syphon water at these height [16]. Water-walking insects have a maximum size of 30 cm because of surface tension effects [17]; the maximum size of prehistoric and extant dragonflies is 1 m and 20 cm, respectively, because of

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