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Large-deflection and post-buckling behavior of slender beam-columns with non-linear end-restraints

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ABSTRACT

The large-deflection analysis and post-buckling behavior of laterally braced or unbraced slender beam-columns of symmetrical cross section subjected to end loads (forces and moments) with both ends partially restrained against rotation, including the effects of out-of-plumbness, are developed in a classical manner. The classical theory of the "Elastica" and the corresponding elliptical functions utilized herein are those presented previously by Aristizabal-Ochoa [1]. The proposed method can be used in the large-deflection analysis and post-buckling behavior of elastic slender beam-columns with rigid, semi-rigid, and simple flexural connections at both ends including linear and non-linear inelastic connections like those that suffer from flexural degradation (such as flexural cracking and elasto-plastic connections) or flexural stiffening. Only bending strains are considered in the proposed analysis. Results from the proposed method are theoretically exact from small to very large curvatures and transverse and longitudinal displacements for laterally braced or unbraced slender beam-columns under bending caused by end loads. The large-deflection analysis and post-buckling behavior of slender beam-columns with both supports partially restrained against rotation and with sway inhibited or uninhibited are complex problems requiring the simultaneous solution of two coupled non-linear equations with elliptical integrals whose unknowns are the limits of the integrals. The validity of the proposed method and equations are verified against solutions available in the technical literature. Three comprehensive examples are included that show the effects of linear and non-linear connections at both ends on the large-deflection analysis and post-buckling behavior of slender beam-columns.

1. Introduction

The non-linear large-deflection analysis and post-buckling behavior of bars and beam-columns are of great importance in physics, structural engineering and engineering mechanics particularly when they are slender and made of high-strength elastic materials like FRP or composites. This problem has been investigated using the exact expression for the curvature in the differential equation of the deflection curve (i.e., the "Elastica" approach, Timoshenko and Gere [2], pp. 76–82) or using the approximate second-order analysis such as the Finite Element Method (FEM) with large deflections and with or without large strains ([3–6], among others).

The exact post-buckling elastic behavior of a perfect beam-column under a gravity load is known; however, its response under other load combinations and the relationships between the applied axial load and the transverse deflections, particularly when the member suffers from imperfections (such as lack of straightness, out-of-plumbness or eccentricity in the applied axial loads) become even more complicated [7]. Khamlichi et al. [8] presented the different formulations to the solution of the large-deflection problem of a perfectly hinged–hinged elastic bar without sidesway under end axial loads, and discussed the effects of the axial strains and shear deformations using the asymptotic expansion technique on the post-buckling behavior.

Aristizabal-Ochoa [9] developed an approximate algorithm based on the classical Timoshenko stability functions for the large-deflection small-strain analysis of beam-columns with semi-rigid connections including the effects of out-of-plumbness, axial strains and lateral bracing. Recently, Aristizabal-Ochoa [1] presented an analytical method for the non-linear large-deflection stability of a slender beam-column of symmetrical cross section with semi-rigid connections (linear and non-linear) and sidesway uninhibited under end

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