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Non-linear flexural–torsional dynamic analysis of beams of arbitrary cross section by BEM

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A B S T R A C T

In this paper, a boundary element method is developed for the non-linear flexural–torsional dynamic analysis of beams of arbitrary, simply or multiply connected, constant cross section, undergoing moderately large deflections and twisting rotations under general boundary conditions, taking into account the effects of rotary and warping inertia. The beam is subjected to the combined action of arbitrarily distributed or concentrated transverse loading in both directions as well as to twisting and/or axial loading. Four boundary value problems are formulated with respect to the transverse displacements, to the axial displacement and to the angle of twist and solved using the Analog Equation Method, a BEM based method. Application of the boundary element technique leads to a system of non-linear coupled Differential–Algebraic Equations (DAE) of motion, which is solved iteratively using the Petzold–Gear Backward Differentiation Formula (BDF), a linear multistep method for differential equations coupled to algebraic equations. The geometric, inertia, torsion and warping constants are evaluated employing the Boundary Element Method. The proposed model takes into account, both the Wagner's coefficients and the shortening effect. Numerical examples are worked out to illustrate the efficiency, wherever possible the accuracy, the range of applications of the developed method as well as the influence of the non-linear effects to the response of the beam.

1. Introduction

In engineering practice the dynamic analysis of beam-like continuous systems is frequently encountered. Such structures often undergo arbitrary external loading, leading to the formulation of the flexural–torsional vibration problem. The complexity of this problem increases significantly in the case the cross section's centroid does not coincide with its shear center (asymmetric beams). Furthermore when arbitrary torsional boundary conditions are applied either at the edges or at any other interior point of a beam due to construction requirements, the beam under the action of general twisting loading is leaded to non-uniform torsion. Moreover, since requirement of weight saving is a major aspect in the design of structures, thin-walled elements of arbitrary cross section and low flexural and/or torsional stiffness are extensively used. Treating displacements and angles of rotation of these elements as being small, leads in many cases to inadequate prediction of the dynamic response; hence the occurring non-linear effects should be taken into account. This can be achieved by retaining the non-linear terms in the strain–displacement relations (finite displacement–small strain theory). When finite displacements are considered, the flexural–torsional dynamic analysis of bars becomes much more complicated, leading to the formulation of coupled and non-linear flexural, torsional and axial equations of motion.

When the displacement components of a member are small, a wide range of linear analysis tools, such as modal analysis, can be used and some analytical results are possible. As these components become larger, the induced geometric non-linearities result in effects that are not observed in linear systems. In such situations the possibility of an analytical solution method is significantly reduced and is restricted to special cases of beam boundary conditions or loading.

During the past few years, the non-linear dynamic analysis of beams undergoing large deflections has received a good amount of attention in the literature. More specifically, Rozmarynowski and Szymczak [1] studied the non-linear free torsional vibrations of axially immovable thin-walled beams with doubly symmetric open cross section, employing the Finite Element Method. In this research only free vibrations are examined; the solution is provided only at points of reversal of motion (not in the time domain), no general axial, torsional or warping boundary conditions (elastic support case) are studied, while some non-linear terms related to the finite twisting rotations as well as the axial inertia term are ignored. Crespo Da Silva [2–3] presented the

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