International Journal of Non-Linear Mechanics 46 (2011) 1116–1127

Contents lists available at ScienceDirect

International Journal of Non-Linear Mechanics

journal homepage: www.elsevier.com/locate/nlm

Flow and heat transfer over a generalized stretching/shrinking wall problem—Exact solutions of the Navier–Stokes equations

Tiegang Fang [a], Shanshan Yao [a], Ioan Pop [b,*]

[a] Mechanical and Aerospace Engineering Department, North Carolina State University, 3246 Engineering Building III—Campus Box 7910, 911 Oval Drive, Raleigh, NC 27695, USA

[b] Faculty of Mathematics, University of Cluj, R-3400 Cluj, CP 253, Romania

ARTICLE INFO

Article history:
Received 2 November 2009
Accepted 5 April 2011
Available online 23 June 2011

Keywords:
Navier–Stokes equation
Similarity equation
Stretching surface
Shrinking sheet
Exact solution
Analytical solution
Heat transfer

ABSTRACT

In this paper, we investigate the steady momentum and heat transfer of a viscous fluid flow over a stretching/shrinking sheet. Exact solutions are presented for the Navier–Stokes equations. The new solutions provide a more general formulation including the linearly stretching and shrinking wall problems as well as the asymptotic suction velocity profiles over a moving plate. Interesting non-linear phenomena are observed in the current results including both exponentially decaying solution and algebraically decaying solution, multiple solutions with infinite number of solutions for the flow field, and velocity overshoot. The energy equation ignoring viscous dissipation is solved exactly and the effects of the mass transfer parameter, the Prandtl number, and the wall stretching/shrinking strength on the temperature profiles and wall heat flux are also presented and discussed. The exact solution of this general flow configuration is a rare case for the Navier–Stokes equation.

1. Introduction

The fluid dynamics over a stretching surface is important in many practical applications, such as extrusion of plastic sheets, paper production, glass blowing, metal spinning, drawing plastic films, the cooling of metallic plates in a cooling bath, polymer sheet extruded continuously from a dye and heat-treated materials that travel between feed and wind-up rolls, to name just a few. Apparently, the quality of the final product depends on the rate of heat and mass transfer between the stretching surface and fluid flow of such processes as explained by Karwe and Jaluria [1]. Since the pioneering study by Crane [2] who presented an exact analytical solution for the steady two-dimensional stretching of a surface in a quiescent fluid with a velocity varying linearly with distance x from a fixed point, many authors have considered various aspects of this problem, such as consideration of mass transfer, power–law variation of the stretching velocity and temperature, magnetic field, application to non-Newtonian fluids, and obtained similarity solutions. Exact solutions for self-similar boundary layer flows induced by a stretching surface with velocity proportional to x^m, where m is a constant, were reported by Banks [3] for an impermeable surface, and by Magyari and Keller [4] for a permeable surface. Liao and Pop [5] solved the case of a linearly stretching surface using the homotopy analytic method (HAM). Carragher and Crane [6], and Grubka and Bobba [7] investigated heat transfer in the above flow in the case when the temperature difference between the surface and the ambient fluid is proportional to a power of distance from the fixed point. Dutta et al. [8] have considered the case of temperature distribution in the flow over a stretching sheet with uniform wall heat flux. Gupta and Gupta [9] analyzed the heat and mass transfer corresponding to the similarity solution for the boundary layers over an isothermal stretching sheet subject to suction or blowing. Bataller [10] performed a numerical analysis in connection with the boundary layer flow and heat transfer of a quiescent fluid over a non-linearly stretching surface. On the other hand, Magyari and Weidman [11] studied the thermal characteristics of the flow over a semi-infinite flat plate driven by a uniform shear in the far field. Similarity solutions of the thermal and momentum boundary layer flow for a power–law shear driven flow over a semi-infinite flat plate has been reported also by Cossali [12] and Fang [13]. Magyari and Keller [14] presented very interesting results for the boundary layer flow and heat transfer characteristics induced by continuous isothermal surfaces stretched with prescribed skin friction. Andersson [15] has considered the slip-flow of a viscous and incompressible fluid past a linearly stretching sheet. Chakrabarti and Gupta [16], and Pop and Na [17] investigated the flow along a permeable stretching sheet under the effect of a constant transverse magnetic field of a Newtonian fluid, while Anderson et al. [18] considered the case of a power–law fluid. Quite recently the flow adjacent to a stretching permeable sheet in a Darcy–Brinkman porous medium has been considered by

* Corresponding author.
E-mail addresses: tfang2@ncsu.edu (T. Fang), popm.ioan@yahoo.co.uk (I. Pop).

0020-7462/$ - see front matter © 2011 Published by Elsevier Ltd.
doi:10.1016/j.ijnonlinmec.2011.04.014