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Testing a fast dynamical indicator: The MEGNO

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ABSTRACT

To investigate non-linear dynamical systems, like for instance artificial satellites, Solar System, exoplanets or galactic models, it is necessary to have at hand several tools, such as a reliable dynamical indicator.

The aim of the present work is to test a relatively new fast indicator, the Mean Exponential Growth factor of Nearby Orbits (MEGNO), since it is becoming a widespread technique for the study of Hamiltonian systems, particularly in the field of dynamical astronomy and astrodynamics, as well as molecular dynamics.

In order to perform this test we make a detailed numerical and statistical study of a sample of orbits in a triaxial galactic system, whose dynamics was investigated by means of the computation of the Finite Time Lyapunov Characteristic Numbers (FT-LCNs) by other authors.

1. Introduction

In the present work we accomplish an exhaustive study of the MEGNO when applied to a given sample of orbits in a triaxial galactic potential studied by [1]. In that work, the authors use a well-known tool, the Lyapunov characteristic numbers (see, e.g., [2]), to identify the character of the selected orbits in order to classify them as regular or chaotic.

The MEGNO is introduced by [3] and, in [4], this technique is formalized and its application extended to discrete Hamiltonian systems like maps; also a generalization of the MEGNO is introduced therein. This tool has become of widespread use for studying several astronomical problems as well as many other Hamiltonian systems (see, e.g., [5–18]).

In [4,19], the MEGNO succeed in furnishing a clear insight of the global structure of the phase space of simple multidimensional Hamiltonian systems, providing a clear picture of the resonant network as well as the regular and chaotic domains.

Herein instead, a far more complex non-linear system is addressed that reproduces many characteristics of real elliptical galaxies, namely, the one introduced by [1]. This model will be used as the scenario for a detailed comparison between the MEGNO and the Lyapunov characteristic numbers and even the fast Lyapunov indicator (FLI) introduced by [20].

There are many efficient dynamical indicators, some of them based on deviation vector(s), for instance, the spectra of stretching numbers, helicity and twist angles, the computation of the alignment indices introduced by Skokos, the Relative finite time Lyapunov Indicator (RLI) and also the average power law exponent (APLE), a technique recently developed [21–24], and others based on spectral analysis, such as [25], the frequency map analysis [26,27], the one due to Sidlichovský and Nesvorný [28], and perhaps the latest one, the FMI (frequency modulation indicator) [29]. However, the present paper is devoted to accomplish a thorough test of the MEGNO, pointing out not only its advantages but its drawbacks as well. Therefore herein we just focus our attention on an exhaustive comparison of the MEGNO against the Lyapunov characteristic numbers (and eventually the FLI), since without any doubt, the latter is the most widespread tool in, at least, the last 40 years, and it is still being used by many authors. On the other hand, as far as we know, a full test of the MEGNO when applied to a non-linear somewhat realistic Hamiltonian system has not been performed yet.

2. The mean exponential growth factor of nearby orbits (MEGNO)

In this section we summarize the main features of the MEGNO (described in detail in [4]). This is an alternative tool to explore the phase space which belongs to the class of the so-called fast indicators.

Let $H(\mathbf{p},\mathbf{q})$ with $\mathbf{p},\mathbf{q} \in \mathbb{R}^N$ be an N-dimensional Hamiltonian that we suppose autonomous just for the sake of simplicity.

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