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# Mode I and mixed mode crack-tip fields in strain gradient plasticity

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#### 1. Introduction

Conventional plasticity predicts that for a quasi-statically growing plane strain crack in an elastic–perfectly plastic solid in mode I, the maximum normal stress ahead of the tip is about three times the initial tensile yield stress,  $\sigma_y$  [1]. Even when strain hardening is taken into account, the normal stress is less than  $4-5\sigma_y$  [2]. The same trend also applies for mixed mode crack propagation.

Experiments on ceramic-metal interfaces have shown guasistatic crack growth of a crack along the interface in the presence of plasticity [3-6]. In these studies, the interface crack remained relatively sharp (not significant blunting) and the estimated stress level needed to result in atomistic/microscopic decohesion is about 10 times the tensile yield stress [7]. However, as pointed out above, this stress level cannot be achieved near the crack-tip according to models based on conventional plasticity. This is not an issue when fracture is associated with void nucleation, growth and coalescence because the ductile fracture mechanism is of the order of 10  $\mu$ m or more and the crack-tip stresses are about 3–5 times the tensile yield stress [8]. However, for atomistic fracture processes (observed in the experiments mentioned) conventional plasticity cannot explain the stress levels necessary for atomistic decohesion. This discrepancy resulted in investigating the role of the plastic strain gradients in elevating the crack-tip stresses in Mode I [7-12]. Wei and Hutchinson [13] and Wei et al. [14] have

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### ABSTRACT

Strain gradients develop near the crack-tip of Mode I or mixed mode cracks. A finite strain version of the phenomenological strain gradient plasticity theory of Fleck–Hutchinson (2001) is used here to quantify the effect of the material length scales on the crack-tip stress field for a sharp stationary crack under Mode I and mixed mode loading. It is found that for material length scales much smaller than the scale of the deformation gradients, the predictions converge to conventional elastic–plastic solutions. For length scales sufficiently large, the predictions converge to elastic solutions. Thus, the range of length scales over which a strain gradient plasticity model is necessary is identified. The role of each of the three material length scales, incorporated in the multiple length scale theory, in altering the near-tip stress field is systematically studied in order to quantify their effect.

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shown a large effect of the strain gradient dependent term on the crack-growth behaviour. Recently, Komaragiri et al. [15] (small strains) and Mikkelsen and Goutianos [16] (finite strains) investigated in detail the role of material length scales on crack-tip stress elevation in Mode I under plane strain using the phenomenological strain gradient plasticity theory of Fleck and Hutchinson [17].

The numerical studies of crack-tip stresses, mentioned above, use different theories of strain gradient plasticity developed over the years after the first strain gradient plasticity theory introduced by [18]. Among the number of theories existing, the more widely used are the theories due to (a) Fleck and Hutchinson [19,17] (higher order theory) and (b) due to Nix and Gao [20] and Gao et al. [21] (lower order theory). A critical assessment of these theories was recently performed by Evans and Hutchinson [22]. It was shown that in the later theory the size effects result in an increase in hardening whereas the initial yield is almost unaffected. The former theory predicts a significant increase in yield strength and little effect on strain hardening. Another basic difference is that the lower order theory cannot be used to simulate passivate interfaces which require the imposition of higher order boundary conditions, i.e. plastic strain=0 along the interface of two different materials. In the higher order theory this is possible due to new quantities mentioned in Section 2.

With these considerations and it's computationally efficiency, the phenomenological theory of Fleck and Hutchinson [17] is used here. As in our previous work for blunted crack-tips [16], a broad range of material parameters (length scales) are systematically varied to determine the range where the gradient plasticity theory is necessary to describe the crack-tip stresses under plane strain conditions. The focus of the current work is on mixed mode

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