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Streamline bifurcations and scaling theory for a multiple-wake model

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ABSTRACT

We investigate the interaction between multiple arrays of (reverse) von Kármán streets as a model for the mid-wake regions produced by schooling fish. There exist configurations where an infinite array of vortex streets is in relative equilibrium, that is, the streets move together with the same translational velocity. We examine the topology of the streamline patterns in a frame moving with the same translational velocity as the streets. Fluid is advected along different paths depending on the distance separating two adjacent streets. When the distance between the streets is large enough, each street behaves as a single von Kármán street and fluid moves globally between two adjacent streets. When the streets get closer to each other, the number of streets that enter into partnership in transporting fluid among themselves increases. This observation motivates a bifurcation analysis which links the distance between streets to the maximum number of streets transporting fluid among themselves. We describe a scaling law relating the number of streets that enter into partnership as a function of the three main parameters associated with the system, two associated with each individual street (determining the aspect ratio of the street), and a third associated with the distance between neighboring streets. In the final section we speculate on the timescale associated with the lifetime of the coherence of this mid-wake scaling regime.

1. Introduction

This paper considers the interaction in the mid-wake region of multiple reverse von Kármán streets as a simple model for the wake interactions associated with schooling fish. In the near wake regime associated with these systems, questions tend to focus on mechanisms of vorticity production and fluid–solid interactions in an unsteady context, whereas in the far-wake regime, questions focus on viscous decay and instability of vortex arrays. In the mid-wake region, however, the vorticity production stage is complete, and the viscous decay and instability of the street is not yet a prominent issue, so questions naturally focus on the interaction of the coherent structures produced by each of the individual wakes. In this regime, we describe a scaling theory that seems to be a new observation associated with multiple von Kármán models. These kinds of models are reminiscent of ones employed in [21,22] in analyzing the hydrodynamic advantages for fish schooling. The focus of [21,22] was on investigating the energy-optimal positioning of an individual fish within the school (the famous diamond pattern) whereas in the present study, we focus on the description of a scaling law relevant for these kinds of wake interactions.

The classical von Kármán vortex street — made up of two rows of evenly spaced point vortices of equal and opposite sign staggered with respect to each other, see [7] — is the canonical 'idealized' vorticity configuration appearing generically in the wake of bluff bodies; see, for example, [23]. While the dynamics and stability of the single street is well-understood (see, for example, [3,15] and references therein), little work is done on the interaction of multiple streets. An infinite array of streets can be viewed, under certain conditions on street alignments, as a special case of doubly periodic vortex lattices whose Hamiltonian dynamics is investigated in [12,18]; see also [4] for an alternative derivation of the equations of motion of doubly periodic rectangular lattices and [13,17] for investigations of singly periodic lattices. Given the results in [18–20], one can argue that there exist configurations where an infinite array of streets is in relative equilibrium. This means that the streamline pattern remains steady in the frame moving with the same translational velocity as the streets. We look at the topology of the streamline patterns, that is to say, the streamline patterns relative to the moving frame, which lends insight into fluid transport through the mid-wake region. Fluid is advected along different paths depending on the distance separating two adjacent streets. Generally, when the distance between the streets is large enough, each street behaves as a single von Kármán street and fluid is transported globally between two adjacent streets. When the streets get closer to each other, the number of streets that enter into partnership in transporting fluid among themselves

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