



Remarks on the solution of extended Stokes' problems

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ABSTRACT

The analytical solutions of first and second Stokes' problems are discussed, for infinite and finite-depth flows of a Newtonian fluid in planar geometries. Problems arising from the motion of the wall as a whole (one-dimensional flows) as well as of only one half of the wall (two-dimensional) are solved and the wall stresses are evaluated.

The solutions are written in real form. In many cases, they improve the ones in literature, leading to simpler mathematical forms of velocities and stresses. The numerical computation of the solutions is performed by using recurrence relations and elementary integrals, in order to avoid the evaluation of integrals of rapidly oscillating functions.

The main physical features of the solutions are also discussed. In particular, the steady-state solutions of the second Stokes' problems are analyzed by separating their "in phase" and "in quadrature" components, with respect to the wall motion. By using this approach, stagnation points have been found in infinite-depth flows.

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1. Introduction

The analytical solution of Stokes problems for a Newtonian fluid in a planar geometry is here revised, by following the seminal paper of Liu [1]. A fluid region is bounded by a rigid wall, which moves with a prescribed velocity having fixed direction, parallel to the wall. The fluid and the wall are at rest at the initial time. By following the literature, wall velocities constant (first Stokes' problem) or periodic (second) in time will be assumed. Moreover, flows in which the wall moves as a whole (one-dimensional) and half wall moves, while the other one is kept fixed, (two-dimensional) will also be investigated. Finally, the depth of the fluid region will be assumed infinite or finite. In these latter kinds of flow, a free surface is assumed to bound the fluid region.

The solution of the first Stokes' problem in an infinite-depth flow has a well known analytical structure, related to the complementary (real) error function. Solutions of the second problem in an infinite-depth flow have been discussed in [2–4] and in many other papers. They are usually written in terms of error functions of complex arguments, because in the corresponding real forms integrands containing oscillatory functions appear, the numerical integration of which can lead to severe errors [5].

Recently, these results have been reconsidered in the framework of two-dimensional flows. In the paper [6] the steady states have been found, while Liu [1] generalizes these solutions, by giving also the transient contributions. The effects of side walls on the Stokes

flow on a planar wall have been recently investigated in [7]. Besides the first and second Stokes problems, the flows induced by a constant accelerating plate and by a plate that applies a constant stress are also investigated. This important paper opens the way to the comparison with experiments, where effects of side walls are rarely negligible.

Despite the subject is a quite old one [8], many issues about analytical solutions and their numerical computation appear to be improved, in particular for two-dimensional flows. The present paper is an attempt to fill some of these lacks. It is organized as follows. In Section 2, the solutions of one-dimensional first and second problems are briefly discussed, then they are extended to the finite-depth case in Section 3. The solution of two-dimensional problems is then faced, for infinite (Section 4) and finite-depth (Section 5) flows. Finally, conclusions are offered in Section 6.

2. One dimensional infinite-depth flows

A Newtonian fluid having kinematical viscosity ν fills the half space $y > 0$, bounded by a solid wall at $y=0$. Initially ($t \leq 0$), fluid and wall are at rest. The wall starts to move at time $t=0^+$ with a given velocity (say q), directed along the axis x . The resulting fluid velocity (u) is assumed to be directed along x and to depend on y and t , only. As well known, this flow is described by Stokes' problem:

$$\begin{cases} \partial_t u = \nu \partial_{yy}^2 u, \\ u(0, t) = q(t), \quad u(+\infty, t) \equiv 0, \\ u(y, 0) \equiv 0, \end{cases}$$

the solution of which is easily found in terms of Laplace transform in time ($q^{(L)}$ and $u^{(L)}$ are the transformed functions of q and u ,

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