



# Conservation Properties of Unsteady Open Channel Flow Equations

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## Abstract

Simulation of unsteady flow in open channels has always been of great interest to the hydraulic engineers. The Saint-Venant equations are usually used for this purpose. The 1-D version of these equations can be written in six different forms by choosing various dependent variables. After clarifying the notion of conservation versus non-conservation forms of these equation sets, the nature of flux vector based on conservation form and the eigenstructure property of Jacobian matrix of flux vector based on non-conservation form are explained in some detail. Shortcomings pertaining to earlier experimental design to confirm the conservation properties of various forms are documented and the role of consistent initial and internal boundary conditions across the discontinuity is highlighted. It is shown that the conservation properties of various forms cannot be uncovered by resorting to non-conservation forms of each equation set. It is recommended that the governing equations be written in conservation form in order to capture the discontinuity under consideration and let the program itself decide which property has to be conserved.

**Keywords:** Saint-Venant, Conservation, Momentum, Energy.

## 1. INTRODUCTION

In 2002, a paper entitled “Conservation-form equations of unsteady open-channel flow” appeared in Journal of Hydraulic Research [1]. The main thesis of the cited paper is to argue that while simulating flow of water in one dimensional open channel, it does matter which combination of state variables must be used to model the process under consideration. We reached the corresponding author through E-mail to share our views [2]. Although more than ten E-mail messages went back and forth between us, we were not convinced and our final conclusion was that the topic deserves further research. We also looked at Science Citation Index (SCI) to learn more about this ongoing research issue and to see if this topic has caught other hydraulic engineers' attention on similar cause. Our search shows that other investigators either accept the paper's findings or refer to the paper in totally different context [3, 4]. It seems that the cited topic has not triggered others' attention. Furthermore, as the paper also touches on conservation aspect of the governing equations, we do believe that the notion of conservation versus non-conservation form of Saint-Venant equations still deserve further clarification and elaboration. As such, this paper is intended to challenge the findings speculated in [1] and clarify other relevant issues on the same topic.

The paper is organized as follow. At first, Some clarification is made on conservation versus non-conservation forms of Saint-Venant equations followed by documenting Saint-Venant equations in both forms using six different combinations of state variables based on geometric ( $A$ ,  $h$ , and  $Z$ ) and kinematic ( $u$  and  $Q$ ) properties of channel cross section, where  $A$  = cross-sectional area;  $h$  = water depth;  $Z$  = water surface elevation;  $u$  = mean velocity at a section, and  $Q$  = volumetric flow rate. Feasibility of converting one form to another is addressed in some detail and the nature of flux vector components for various state variable combinations is highlighted. Next, the experimental design stipulated in [1] is reviewed, challenged and another version of experimental design with different perspective is provided. Results obtained from this new experimental design and the old one are compared and contrasted with those obtained in [1]. Conclusions are summarized in the last section.

## 2. CONSERVATION VERSUS NON-CONSERVATION FORMS OF SAINT-VENANT EQUATIONS

### 2.1. CONSERVATION FORM

In a nutshell, the resulting equations will be in conservation form when Reynolds Transport Theorem (RTT) is implemented for a fixed control volume in space where the fluid moves through it [5]. When the size of