Elastic Multilayer Analysis Using Extrapolating Oscillatory Integration

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Abstract

In classical mechanics, solutions to various solid mechanics problems have been very well established. A famous one is the Burmister's solution for the case of a uniformly distributed circular vertical load acting on the surface of a three-layered system. In practical problems like pavement and foundation analysis, accurate and efficient solutions for multilayer systems are needed. Solutions presented in this paper make use of Hankel transform. In Hankel transform a semi-infinite integral is confronted that must be calculated using numerical methods. In this research, codes developed in *Mathematica* are used to assess the efficiency of extrapolating oscillatory integration method for this special case. The criteria used for assessing the efficiency of this method is the number of integrand evaluation points to satisfy the specified precision. Problems were found only in the vicinity of the surface.

Keywords: Elastic Multilayer System, Hankel Transform, Extrapolating Oscillatory Integration

1. INTRODUCTION

One of the best-known problems in solid mechanics is the case of a concentrated vertical load applied to the surface of a semi-infinite body and Boussinesq has achieved the solution of this problem. By integrating this solution in the range of a circle, solutions for the case of a cylindrical load can be obtained. However, in practical problems like road or airport pavement analysis, solutions to a multilayer system are needed. These solutions must be accurate and simple. The simplest way to characterize a flexible pavement under a tire load is to consider it as a surface composed of an elastic multilayer system on which a cylindrical load acts.

Solutions presented in this paper make use of Hankel transform. The semi-infinite integral in the Hankel transform is calculated using Extrapolating Oscillatory integration technique. Number of integrand evaluation points is considered as a criteria for assessing the efficiency of the numerical integration at different points. For a given integral, the integration technique which achieves a result that satisfies the specified precision with the smallest number of integrand evaluation points is considered to be more efficient.

The multilayer elastic theory developed by Burmister [1] has been employed in many researches to analyze pavements. With the advent of modern computers, the theory can be applied to a multilayered system with any number of layers and Hankel transform can be used to achieve the solutions [2]. Mania and Matsui have employed a Richardson's extrapolation to improve the convergence [3]. Erlingsson and Ahmed have developed a method to optimize the calculation of stresses and displacements [4].

2. PROBLEM MODELLING

Figure 1 shows the geometry and loading conditions of an elastic multilayer system subjected to a cylindrical load. E, ν and h are the elastic modulus, Poisson's ratio and thicknesses of layers respectively; q is the amplitude and a is the radius of cylindrical loading.