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Governing Equations of Micro-scale Plate in terms of Displacement Potential Functions

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Abstract
The behavior of solids can be represented by variety of continuum theories. The deformation behavior of materials in the micron scale has been experimentally shown to be size dependent. Therefore, a number of theories containing material length scale parameters and extra freedoms have been presented to solve this defect. For example, Cosserat elastisity allows the points in the continuum to rotate as well as translate, another proposed theory is Couple stress theory which is utilized in this paper.

In this study, by using the classical form of Lekhnitskii-Hu-Nowacki displacement potential functions, the governing equations of couple stress theory for isotropic micro or nano materials are obtained. These equations include three Partial Differential Equations of sixth order. Solution of the governing equations have shown that the effect of lenghth paremeter is ommited. Due to this defect, the Lekhnitskii-Hu-Nowacki displacement potential functions are modified for micro or nano scale and new potential function is presented. Based on modified displacement potential functions, governing equations are solve for simply supported rectangular plate for micro or nano scale. Comparison of obtained results using proposed method in this research with other works shows good agreement between results.

Keywords: Couple stress, micro-scale, microrotation, governing equations, length scale parameter

1. INTRODUCTION

The Classical Theory of Elasticity which is now used in continuum mechanics is inadequate for considering size dependency in micro and nano-scales. Actually, difference between classical theory and experimentally results in micro scales reveals it. Therefore, a number of theories containing material length scale parameters have been presented to solve this defect. W.Voigt (1887), as a pioneer in researches, proposed that for an element besides the force stresses σ_{ij}, the couple stresses m_{ij} should be considered . However, the Francois and Eugene Cosserat brothers (1909) developed the microstructure elasticity by assumption that each element during the deformation not only displaces but also rotates independent of the classical macrorotation, which means they considered an extra freedom for an element .In addition, they assumed u and φ vectors, respectively displacement function and rotation function, are independent [1]. Later, the researches revived by Toupin (1962), Mindlin (1962), Koiter (1964), Nowacki(1986), and they considered the second gradient of deformation in couple stress elastic materials. According to Mindlin formulation, in couple stress theory there are two material constants η and η'. In hindsight, Hadjesfandiari& Dargush have indicated that they are not independent and $\eta' = -\eta$ [2], while Yang et al [3] have shown that the number of required material length scale parameters is only one in the new modified couple stress framework, instead of two in the classical couple stress theory.

For classification, one branch revives the idea of microrotation and is called micropolar theory, the other main branch, labeled second gradient theories, avoids the idea of microrotation by introducing the gradients of strain or rotation [2]. Micropolar is one of the microstructure continuum mechanic theories, in which Eringen suggested the location vector and a rigid vector representing its inner rotation define the motion of a particle. The particle's motion is characterized by the changes of the vectors [3]. In fact, micropolar has six degrees of freedom, three translation freedoms related to macrostructure and three rotation freedoms related on microstructure .While, in the couple stress theory suggested by Toupin, it was assumed that the rotation of microstructure is equal to that of macrorotation [4].

Here we use the couple stress theory developed by Hadjesfandiari& Dargush [5]. Based on the couple stress theory and Lekhnitskii-Hu-Nowacki displacement potential functions which successfully have been considered for bending analysis of isotropic simply supported thick rectangular plate by Nematzadeh and Eskandari-Ghadi and Navayi Neya [6], we obtained the governing equations of isotropic medium. Then the obtained governing equations are solved and obtained results are compared with other works.