

# Exact supercritical vibration of traveling orthotropic plates using dynamic stiffness method

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#### Abstract

Axially traveling plates are used in modeling the transverse vibration of a variety of physical systems, e.g. band saw blades, power transmission belts, steel plates in galvanizing line, paper handling and textile manufacturing. In this paper the Dynamic Stiffness Method (DSM) is employed to analyze the free vibration of traveling orthotropic plates in supercritical speeds. Critical speeds are defined as those axial speeds where the system vibration has a vanishing eigenvalue and is subject to a buckling instability. Transport speeds above the lowest critical speed are termed supercritical. Based on classical plate theory, a dynamic stiffness matrix is formed by frequency dependent shape functions which are exact solutions of the governing differential equations. It eliminates spatial discretization error and is capable of predicting several natural modes by means of a small number of degrees of freedom. This stiffness matrix is a complex matrix and contains axial speed, in-plane forces and free vibration eigenvalues which are generally complex. To extract these complex eigenvalues, an algorithm is required to search in the two-dimensional plane of complex numbers. In the current study, use is made of such an algorithm. With a few test cases, the reliability of the formulation and the solution procedure is shown.

Keywords: Dynamic Stiffness Method, Axially moving, Orthotropic plate, Supercritical speed

### 1. Introduction

Axially moving plates are used in modeling the transverse vibration of a variety of physical systems, e.g. band saw blades, power transmission belts, steel plates in galvanizing line, paper handling and textile manufacturing.

Because of their axially speed, moving materials experience a Coriolis acceleration component which renders such systems gyroscopic. In a certain critical speed, first natural frequency of the gyroscopic system vanishes and the structure experience severe vibrations and bifurcation instability. In the speeds above the critical speed, termed supercritical speeds, the structure may experience flutter instability or become stable again. Since the transverse vibration of the moving material often becomes a serious problem in achieving good quality in axially traveling systems, accurate prediction of dynamic characteristics of such systems is requisite for the optimal design of them.

Prior research on axially moving plates has often provided a thorough understanding of the natural frequencies, eigenfunctions, and response of the plate moving below critical speed. In 1968, Soler [1] used a simple bending-torsion model for a wide band moving with constant speed. Ulsoy and Mote [2] is likely to be the first who used a two-dimensional plate model for a wide bandsaw blade. They applied the classical Ritz method and the finite element-Ritz method. Lin and Mote [3] used the von Karman nonlinear plate theory to investigate the large equilibrium displacement and stress distribution of a web under transverse loading.

Lin [4] studied the linear vibrations of travelling plates; beam and string theories were compared with plate theory: the beam model overestimates and string model underestimates the critical speed. Moreover, it was found that, in the super-critical speed range, the trivial equilibrium position is always unstable. Such results highlight the main discrepancies between plates and beam/string theories, which are particularly important for small width–length ratios and high flexural stiffness.

Wang [5] developed a mixed finite element formulation for a moving orthotropic plate based on the Mindlin– Reissner plate model. Damaren and Langoc [6] applied the Rayleigh–Ritz method to formulate the discreteparameter motion equations purely for active control application.