## **Analyzing Innermost Runtime Complexity of Term Rewriting by Dependency Pairs**

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**Abstract** We present a modular framework to analyze the innermost runtime complexity of term rewrite systems automatically. Our method is based on the dependency pair framework for termination analysis. In contrast to previous work, we developed a *direct* adaptation of successful termination techniques from the dependency pair framework in order to use them for complexity analysis. By extensive experimental results, we demonstrate the power of our method compared to existing techniques.

**Keywords** Complexity analysis • Term rewriting • Termination analysis • Dependency pairs

## **1** Introduction

In practice, one is often not only interested in analyzing the termination of programs, but one also wants to check whether algorithms terminate in *reasonable* (e.g., polynomial) *time*. While termination of term rewrite systems (TRSs) is well studied, only recently first results were obtained which adapt termination techniques in order to obtain polynomial complexity bounds automatically, e.g., [2–5, 7, 10, 17–19, 22–24, 26, 29, 30]. Here, [3, 17–19] consider the *dependency pair (DP) method* 

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