

Reducing Higher-Order Theorem Proving to a Sequence of SAT Problems

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Abstract We describe a complete theorem proving procedure for higher-order logic that uses SAT-solving to do much of the heavy lifting. The theoretical basis for the procedure is a complete, cut-free, ground refutation calculus that incorporates a restriction on instantiations. The refined nature of the calculus makes it conceivable that one can search in the ground calculus itself, obtaining a complete procedure without resorting to meta-variables and a higher-order lifting lemma. Once one commits to searching in a ground calculus, a natural next step is to consider ground formulas as propositional literals and the rules of the calculus as propositional clauses relating the literals. With this view in mind, we describe a theorem proving procedure that primarily generates relevant formulas along with their corresponding propositional clauses. The procedure terminates when the set of propositional clauses is unsatisfiable. We prove soundness and completeness of the procedure. The procedure has been implemented in a new higher-order theorem prover, Satallax, which makes use of the SAT-solver MiniSat. We also describe the implementation and give several examples. Finally, we include experimental results of Satallax on the higher-order part of the TPTP library.

Keywords Higher-order logic • Simple type theory • Higher-order theorem proving • Abstract consistency • SAT solving

1 Introduction

There are a number of distinct aspects of automated theorem proving. First, there is the usual combinatorial explosion already associated with search in the propositional

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