Non-Linear Rewrite Closure and Weak Normalization

**Carles Creus · Guillem Godoy · Francesc Massanes ·
Ashish Tiwari**

Abstract A rewrite closure is an extension of a term rewrite system with new
rules, usually deduced by transitivity. Rewrite closures have the nice property
that all rewrite derivations can be transformed into derivations of a simple form.
This property has been useful for proving decidability results in term rewriting.
Unfortunately, when the term rewrite system is not linear, the construction of a
rewrite closure is quite challenging. In this paper, we construct a rewrite closure for
term rewrite systems that satisfy two properties: the right-hand side term in each
rewrite rule contains no repeated variable (right-linear) and contains no variable
occurring at depth greater than one (right-shallow). The left-hand side term is
unrestricted, and in particular, it may be non-linear. As a consequence of the rewrite
closure construction, we are able to prove decidability of the weak normalization
problem for right-linear right-shallow term rewrite systems. Proving this result also
requires tree automata theory. We use the fact that right-shallow right-linear term
rewrite systems are regularity preserving. Moreover, their set of normal forms can be

C. Creus · G. Godoy (✉) · F. Massanes
Universitat Politècnica de Catalunya, Omega building,
Jordi Girona 1-3, Barcelona 08034, Spain
e-mail: ggodoy@lsi.upc.edu

C. Creus
e-mail: ccreuslopez@gmail.com

F. Massanes
e-mail: f.massanes@gmail.com

A. Tiwari
SRI International, Menlo Park, CA 94025, USA
e-mail: tiwari@csl.sri.com