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Formal Power Series

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Abstract We present a formalization of the topological ring of formal power series in Isabelle/HOL. We also formalize formal derivatives, division, radicals, composition and reverses. As an application, we show how formal elementary and hypergeometric series yield elegant proofs for some combinatorial identities. We easily derive a basic theory of polynomials. Then, using a generic formalization of the fraction field of an integral domain, we obtain formal Laurent series and rational functions for free.

Keywords Formalization of mathematics · Theorem proving · Formal power series · Generating functions

1 Introduction

The theory of formal power series (*fps*) lays the foundation for substantial parts of combinatorics and real and complex analysis. This paper presents a formalization of this theory in the proof assistant Isabelle/HOL. The interest in such a formalization is not only due to the importance of the theory within the body of formalized mathematics but also to its connections to polynomials and formal Laurent series. Since *fps* are more general than polynomials, we obtain these in a very simple manner. The generic construction of the fraction field over any integral domain ultimately yields formal Laurent series. Formalizing *fps* is hence a strategic choice when building a mathematical library in a proof assistant. One further benefit is the

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