

Tractable Extensions of the Description Logic \mathcal{EL} with Numerical Datatypes

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Abstract We consider extensions of the lightweight description logic (DL) \mathcal{EL} with numerical datatypes such as naturals, integers, rationals and reals equipped with relations such as equality and inequalities. It is well-known that the main reasoning problems for such DLs are decidable in polynomial time provided that the datatypes enjoy the so-called convexity property. Unfortunately many combinations of the numerical relations violate convexity, which makes the usage of these datatypes rather limited in practice. In this paper, we make a more fine-grained complexity analysis of these DLs by considering restrictions not only on the kinds of relations that can be used in ontologies but also on their occurrences, such as allowing certain relations to appear only on the left-hand side of the axioms. To this end, we introduce a notion of safety for a numerical datatype with restrictions (NDR) which guarantees tractability, extend the \mathcal{EL} reasoning algorithm to these cases, and provide a complete classification of safe NDRs for natural numbers, integers, rationals and reals.

Keywords Description logic · Computational complexity · Datatypes

1 Introduction and Motivation

Description logics (DLs) [1] provide a logical foundation for modern ontology languages such as OWL¹ and OWL 2 [2]. \mathcal{EL}^{++} [3] is a lightweight DL for which

¹<http://www.w3.org/2004/OWL>

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