

# Proof Pearl: a Formal Proof of Higman's Lemma in ACL2

Francisco Jesús Martín-Mateos · José Luis Ruiz-Reina ·  
José Antonio Alonso · María José Hidalgo

Received: 16 October 2009 / Accepted: 24 May 2010 / Published online: 18 June 2010  
© Springer Science+Business Media B.V. 2010

**Abstract** Higman's lemma is an important result in infinitary combinatorics, which has been formalized in several theorem provers. In this paper we present a formalization and proof of Higman's Lemma in the ACL2 theorem prover. Our formalization is based on a proof by Murthy and Russell, where the key termination argument is justified by the multiset relation induced by a well-founded relation. To our knowledge, this is the first mechanization of this proof.

**Keywords** Higman's lemma · Formal proofs · ACL2

## 1 Introduction

Higman's lemma [7] is a result in the field of combinatorics, stating well-quasi-orderness of a certain embedding relation on finite strings over a well-quasi-ordered alphabet. It provides a criterion for proving termination of string rewrite systems and it is a particular case of Kruskal's tree theorem, which plays a fundamental role in the proof of well-foundedness of certain orderings used to show termination of term rewriting systems [1].

---

F. J. Martín-Mateos · J. L. Ruiz-Reina (✉) · J. A. Alonso · M. J. Hidalgo  
Computational Logic Group, Department of Computer Science and Artificial Intelligence,  
University of Seville, E.T.S.I. Informática, Avda. Reina Mercedes, s/n. 41012 Sevilla, Spain  
e-mail: jruiz@us.es

F. J. Martín-Mateos  
e-mail: fjesus@us.es

J. A. Alonso  
e-mail: jalonso@us.es

M. J. Hidalgo  
e-mail: mjoseh@us.es