

Nested Abstract Syntax in Coq

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Abstract We illustrate *Nested Abstract Syntax* as a high-level alternative representation of languages with binding constructs, based on nested datatypes. Our running example is a partial solution in the Coq proof assistant to the POPLmark Challenge. The resulting formalization is very compact and does not require any extra library or special logical apparatus. Along the way, we propose an original, high-level perspective on environments.

Keywords POPLmark · Abstract syntax · Semantics · Nested datatypes · Coq · System $F_{<}$

1 Introduction

We present a partial solution to the POPLmark Challenge [2] in the Coq proof assistant. Our formalization addresses part (1A) of the Challenge, concerning the transitivity property of the subtyping relation of System $F_{<}$. Our solution is distinguished by the use of nested datatypes to encode the types of System $F_{<}$. Accordingly, we propose an original “higher-order” encoding of environments, as functions on the varying set of variables.

Nested datatypes, also called heterogeneous datatypes, have been popularized by the work of Bird and Meertens [4]. It is a general technique to enforce certain invariants through typing, which allows a high-level and perfectly natural representation of variable binding, as was experienced already ten years ago [1, 5]. We

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