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# Optimal PMU placement method for complete topological and numerical observability of power system

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#### ARTICLE INFO

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Keywords: Phasor measurement unit (PMU) Optimal PMU placement Numerical observability Measurement Jacobian Rank deficiency This paper presents a new method of optimal PMU placement (OPP) for complete power system observability. A two-stage PMU placement method is proposed, where stage-1 finds out the minimum number of PMUs required to make the power system *topologically observable* and stage-2 is proposed to check if the resulted PMU placement (from stage-1) leads to a full ranked measurement Jacobian. In case the PMUs placed, ensuring topological observability in stage-1, do not lead to the Jacobian of full rank, a sequential elimination algorithm (SEA) is proposed in stage-2 to find the optimal locations of additional PMUs, required to be placed to make the system *numerically observable* as well. The proposed method is tested on three systems and the results are compared with three other topological observability and also demonstrate the need of using stage-2 analysis along with the topological observability based PMU placement methods.

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#### 1. Introduction

State estimator (SE) is an essential tool for the real time monitoring of the power systems. Conventional state estimators use a set of measurements consisting of busbar voltages, real and reactive power flows, power injections in order to estimate busbar voltage phasors in the system. Until recently, these measurements were obtained only through supervisory control and data acquisition (SCADA) systems and it was not possible to measure phase angles of the busbar voltages in real time. With the advent of wide area measurement system (WAMS) [1-3], employing phasor measurement units (PMUs), this problem has been alleviated. Time synchronization of the voltage and current phasors at different locations is achieved through global positioning system (GPS). A minimum number of appropriately distributed PMUs are needed in order to carry out the power system state estimation and the complete observability of the system is a prerequisite to the state estimation. The observability of power system has been viewed in terms of its numerical observability as well as topological observability [4].

Numerical observability based approaches utilize the information (or gain) matrix or the measurement Jacobian. When the

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measurement Jacobian is of full rank, the network is said to be numerically observable. Many OPP techniques, based on this concept, have been devised. Simulated annealing [5], tabu search [6], and genetic algorithm [7] have been used to find the optimal PMU locations in the system. However, these methods are iterative in nature and involve extensive matrix manipulations and are, therefore, computationally extensive. On the other hand, topological observability based approaches focus on the placement of measurements to obtain an observable system utilizing the graph concept. A few methods, based on this concept, are depth first search [8], spanning tree based method [9,10] and integer linear programming based methods [11]. Another PMU placement method suggested by Rakpenthai et al. [12] uses the condition number of the normalized measurement matrix as a criterion for selecting candidate solutions, along with binary integer programming to find the PMU locations. Recently, Chakrabarti and Kyriakides [13] have suggested a method for the OPP based on the exhaustive binary search. In [14], the authors have suggested a binary particle swarm optimization (BPSO) technique to achieve dual objectives: to minimize the number of PMUs, required for making the system fully observable, and to maximize the measurement redundancy.

A literature survey reveals that the most of the previous work has proposed topological observability based PMU placement methods, which may not always ensure total system observability required for successful execution of the SE. Hence, the main motivation in this work is to ensure the numerical observability of the system along with the topological observability while optimally placing the PMUs in the power system network.

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