



## Loss sensitivity formulas by adjoint networks

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### ABSTRACT

This paper presents a new set of formulas for computing power system loss sensitivities with respect to power system parameters. They are derived in the context of Tellegen's theorem and the concept of adjoint networks. The formulas are explicit, showing the combination of the power network quantities and the adjoint networks quantities. The formulas deal with branch-related and node-related parameters with equal ease. Data and numerical results for the adjoint quantities and for the sensitivities are presented.

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## 1. Introduction

### 1.1. Objective

The objective of this paper is to present new formulas for computing sensitivities of power system losses with respect to any power system parameter. The formulas make all kinds of power loss sensitivities readily available and with equal handling – sensitivities with respect to node quantities such as loads (active and reactive powers) or generator parameters (active power and voltage set up) are handled in the same manner as those sensitivities with respect to network changes as expressed by changes in resistances and/or reactances of lines.

The reason for this equal handling of node and branch quantities lies in the basis for the formulas derived: the formulas presented in this paper are derived from Tellegen's theorem and the concept of adjoint network for incremented networks, rather than from the conventional power flow equations. [For Tellegen's theorem all quantities involved are expressed as branch quantities]

This paper presents the sensitivity formulas, their derivation using a very special adjoint network, and a numerical example showing values for key steps in the computation of these sensitivities to help the reader replicate the results.

### 1.2. Basis

The basis for the work presented in this paper is Tellegen's Theorem (TT) [1,2]. Consider a power system network  $N$  where  $V_k$  is

the voltage across branch  $k \in K$ , and where  $K$  is the index set for all network branches. Consider network  $\hat{N}$ , a network assumed as adjoint to the power system network, where  $\hat{I}_k$  is the corresponding current through branch  $k$ . In its strong form, TT can be stated as

$$\sum_{k \in K} V_k \hat{I}_k = 0 \quad (1)$$

The present paper focuses on determining the sensitivities of power system losses (first-order incremental losses) with respect to power system parameters (first-order incremental changes). The form of TT chosen is the quasi-power difference form for incremental networks:

$$\sum_{k \in K} \hat{I}_k \Delta V_k - \hat{V}_k \Delta I_k = 0 \quad (2)$$

Eq. (2) is a complex equation. Losses are a real quantity. We will use the real part of (2):

$$\sum_{k \in K} \text{Re}\{\hat{I}_k \Delta V_k - \hat{V}_k \Delta I_k\} = 0 \quad (3)$$

### 1.3. Related work

The computation of power system loss sensitivities has long been of importance for power system planning and operation. There are two most successful methods: the so-called B method, based on a DC analysis; and the method based on the Jacobian of the AC power flow equation [3–5]. The subject of power system loss sensitivity has been addressed in recent papers in the context of transaction evaluation, voltage stability, and sensitivity analysis and loss minimization, e.g. [6–8]. The focus of this paper is not on

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