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Analytical Solution of Wave Shoaling Based on Cnoidal Wave Theory

[*Hadi . Sadeghian* هادي صادقيان]

[*Peyman . Badiei* پيمان بديعي]

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1- Introduction

Cnoidal wave theory which is derived from Korteweg and de Vries equation for waves with constant form, describe the wave characteristics in terms of Jacobian elliptical functions. Small-amplitude and solitary wave theories are respectively the deep and shallow water limits of this theory [1]. An advantage of cnoidal wave theory is that it gives an analytical solution for weakly nonlinear Boussinesq waves of constant form. It is, therefore, possible to derive some of the principal properties of Boussinesq waves by applying this theory. Unlike the linear waves, properties such as wave length and celerity are not only dependent on water depth and wave period but also on wave height [2]. In this paper, analytical solution of cnoidal wave height variation due to shoaling is formulated. In other words, shoaling of cnoidal waves propagating normal to a gently sloping beach with straight and parallel bottom contours is solved analytically based on the conservation of energy equation in the absence of dissipation [3 & 4]. Analytical solution of cnoidal and linear shoaling waves are compared for a special test case which indicates an about 5% increase of wave height at a given depth, when cnoidal theory is applied. The result of analytical solution is also compared with the numerical solution of Boussinesq equations using MIKE 21-BW model, in which acceptable agreement is evident.

2- Governing equations of cnoidal wave theory

Based on the definition sketch shown in (Fig.1), wave characteristics, such as water surface profile and wave length, are formulated in accordance with cnoidal wave theory as follows.

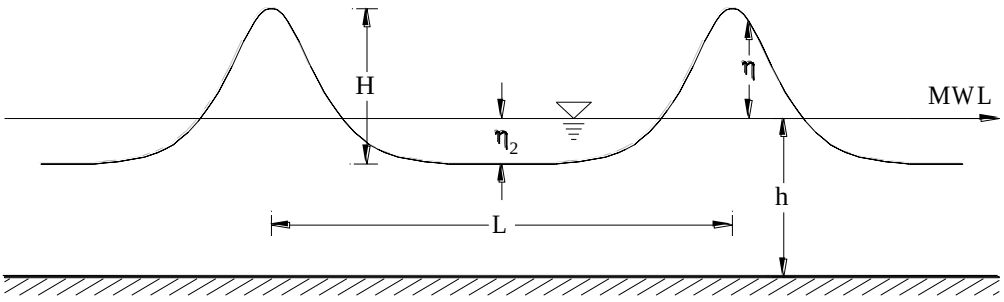

Fig.1(Definition sketch for cnoidal wave