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ORIGINAL ARTICLE

Friedmann universe as a limit of dust shell model

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Abstract We generalize the equations of motion of a shell for a sequence of thin shells and derive the general form of the expansion law of the dust shell model which represents the FRW universe when we take an appropriate limit. The three Friedmann model for the three possible values of $k: 0, \pm 1$ are satisfied.

Keywords Cosmology: theory · Early universe · Large scale structure of the universe

1 Introduction

The standard big bang model is based on the assumption of the homogeneous and isotropic distribution of matter and radiation. This assumption leads to the Robertson-Walker-Friedmann universe model through the Einstein equation. This standard model has succeeded in explaining various important observational facts: Hubble's expansion law, the content of light elements, and the existence of the cosmic microwave background radiation Lachieze-Rey and Gunzig (1999). In fact, the deviation of our universe from a homogeneous and isotropic space is as small as $\sim 10^{-5}$ at the stage of decoupling, Smoot et al. (1992) . Thus our universe is well approximated by a FRW model before this stage. On the other hand, the present universe is highly inhomogeneous on small scale. As an inhomogeneous model, we take spherically symmetric dust shell, in which light rays travel from each dust shell to an observer at the center.

Our main goal in this paper is to clarify under which condition a dust shell model behaves like a FRW universe. The organization of this paper is as follows. In the next section, we give the general formalism. We summarize the basic equations for the dynamics of a one shell and generalize these equations of motion for a sequence of shells in Sect. 3. This is an extension of the treatment in paper Eid and Langer (2000). In Sect. 4 we compared a dust shell model with a FRW universe. Finally Sect. 5 is devoted to the conclusions.

2 Formalism

Our approach is similar to that used by Israel (1966) for studying the collapse of spherical shells. The time-like hypersurface Σ, which divides the Riemannian space-time $\mathcal{M}$ into two regions, $\mathcal{M}^-$ and $\mathcal{M}^+$, represents the history of a thin spherical shell of matter in general. The regions $\mathcal{M}^-$ and $\mathcal{M}^+$ are covered by the mutually independent coordinate systems X^α_- and X^α_+. The hypersurface Σ represents the boundary of $\mathcal{M}^-$ and $\mathcal{M}^+$ respectively; consequently the intrinsic geometry of Σ induced by the metrics of $\mathcal{M}^-$ and $\mathcal{M}^+$ must be the same. Let Σ be parameterized by intrinsic coordinates ξ^a,

$$X^\alpha_\pm = X^\alpha_\pm(\xi^a).$$

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