

# Basic thermodynamic parameters of a black hole resulting from a Yukawa type of correction to the metric

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**Abstract** There has been a renewed interest in the recent years in the possibility of deviations from the predictions of Newton’s “inverse-square law” of universal gravitation. One of the reasons for renewing this interest lies in various theoretical attempts to construct a unified elementary particle theory, in which there is a natural prediction of new forces over macroscopic distances. Therefore the existence of such a force would only coexist with gravity, and in principle could only be detected as a deviation from the inverse square law, or in the “universality of free fall” experiments. New experimental techniques such that of Sagnac interferometry can help explore the range of the Yukawa correction  $\lambda \geq 10^{14}$  m where such forces might be present. It may be, that future space missions might be operating in this range which has been unexplored for very long time. In this paper we derive the basic thermodynamic parameters of such a Yukawa stationary spherically symmetric black hole. First, the expression for the event horizon of such a black hole is derived, with the help of which the temperature, entropy and heat capacity of this particular black hole are obtained. We have also obtained analytical expressions for the change of mass of such black hole, and also its corresponding evaporation time.

**Keywords** Yukawa potential · Yukawa black hole · Black hole temperature · Black hole entropy · Black hole heat capacity · Black hole evaporation time · Gravity theories

## 1 Introduction

Although the accuracy and prediction power of Newton’s gravitational theory has been proved in a lot of cases, observations do not always match the inverse square force law. Therefore, many of the present theories of gravitation such as supergravity unified gauge theories, string theory, supersymmetry and higher derivative gravity theories predict forces coupled to gravitation. Predictions of new light and massless elementary particles have drawn a considerable attention for possible deviations from the Newtonian law of universal gravitation. It has also been theoretically predicted that these deviations can be described by a Yukawa type of potential. These forces are expected to be significant for pairs of celestial bodies that lie in a mutual distance greater than  $10^{10}$  m, or for elementary massive particles separated by distances in a submillimeter range. While some experimental work has already been developed for the second case (see, e.g., Chiaverini et al. 2003; Hoyle et al. 2004; Smullin et al. 2005), the first domain remains almost unexplored (Camacho 2004). Experimental techniques, such as the Sagnac interferometry, and future space missions may be of much help in exploring this range.

The above mentioned non-Newtonian forces are expressed by an additional term, which must be incorporated into the original  $1/r$  Newtonian potential. For instance, Newton himself (in his unpublished work, Portsmouth Collection, 1888) has proposed a modified potential of the form  $a/r + b/r^2$  with  $a, b$  positive constants. Maneff

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