9th International Congress on Civil Engineering, May 8-10, 2012
Isfahan University of Technology (IUT), Isfahan, Iran

Constitutive Equations for the Elastic Behavior of Granular Soils with Thermodynamics Considerations

Ali Golchin[1], Ali Lashkari[1]

1- Department of Civil & Environmental Engineering, Shiraz University of Technology, Shiraz, Iran

lashkari@sutech.ac.ir

Abstract

Formulations of four elastic models of granular soils belonging to two different families of the elasticity theory, hypoelastcity and hyperelasticity, are described. To investigate the impact of the elastic response on predictions of elasto-plastic models, the introduced elasticity models are implemented within an advanced elasto-plastic bounding surface constitutive model. Then, the elasto-plastic models with the same elements for the plastic part and different elasticity models are applied for simulation of undrained tests. The most favorable predictions are obtained from the model with anisotropic hypoelasticity, whereas predictions by the model with isotropic elasticity were found undesirable. Finally, predictions of the models employing hyperelasticity were slightly better than those of the isotropic hypoelastic model.

Keywords: Hypoelasticity, Hyperelasticity, Gibbs free energy, Plasticity theory, Liquefaction.

1. INTRODUCTION

The mechanical behavior of granular soils is considered complex when it is compared to those of the other engineering materials (like steel), because granular soils exhibit shear-volume strain coupling (say dilatation) when they are subjected to shear stress. In addition, domain of pure elasticity in granular soils is quite small and is restricted to very small shear strains ($\approx 10^{-5}$). As a result, the mechanical response of granular soils is mostly elasto-plastic which implies that even small shear actions induce both elastic (reversible) and plastic (irreversible) strains. Moreover, it is observed that sands behavior is highly dependent on internal variables describing the mechanical state of the medium. For example, experimental studies using resonant column and bender element have revealed that sands elastic shear and bulk moduli are highly influenced by the current state of density as well as the current value of mean principal effective stress. The last, but not the less important factor affecting the elastic moduli is stress-induced anisotropy (e.g., [1,2]). For granular soils, the elastic shear modulus is usually defined by (e.g., [1]):

$$G = G_0 \, p_{ref} \, F(e) \left(\frac{p}{p_{ref}} \right)^n \tag{1}$$

where, G_0 is a dimensionless material parameter, p_{ref} is a reference pressure that is normally taken as the atmospheric pressure (=101 kPa). F(e) is a function of void ratio, e, imposing the influence of density on G. The term $(p/p_{ref})^n$ is introduced to consider the influence of mean principal effective stress, p, on shear modulus. n=1/2 has been suggested in many references based on the result of resonant column tests [1]. Adopting a constant Poisson's ratio (ν), the elastic bulk modulus in isotropic soils is calculated by:

$$K = \frac{2}{3} G \left(\frac{1+\nu}{1-2\nu} \right) \tag{2}$$

It is widely accepted that sands elastic moduli are functions of mean principal effective stress. This particular behavior is a unique aspect of the geomaterials. From a historical view point, Hertz [3] was the first one who proved the pressure dependency of the elastic response of the granular media. Using the elasticity theory, Hertz [3] studied the stiffness of identical smooth spheres in contact with each other. He derived an analytical expression for such media in which the elastic stiffness is a function of $p^{1/3}$. Recently, Jenkins & Strack [4] used a rigorous approach for the isotropic compression of a triaxial sample of a random assembly of identical spheres and obtained a mathematical expression similar to that of Hertz [3] in which the elastic bulk modulus is dependent on $p^{1/3}$. Contrary to the analytical findings of Hertz [3] and Jenkins & Strack [4], a large number of experimental evidences under very low strain levels indicate that the granular soils moduli depend on $p^{1/2}$. Goddard [5] considered this issue and suggested that the assumption of smooth contact between spheres may be unrealistic. He considered a pointed conical asperity in contact zone and obtained the following expression for the bulk modulus of granular masses: