7th National Congress on Civil Engineering, 7-8 May 2013
University of Sistan and Baluchestan, Zahedan, Iran

Closed-form analysis of free vibration of a two-span beam in the frequency domain

S. M. Hozhabrossadati[1], A. Aftabi Sani[2]

1,2- Department of Civil Engineering, Mashhad Branch, Islamic Azad University, Mashhad, Iran

sm.hozhabrossadati@yahoo.com

Abstract

The purpose of this study is to deal with the free vibration of a two-span Euler-Bernoulli beam. The ratios of flexural rigidity and mass per unit length of two spans of beam are different and variable. The governing eigenvalue problem of system, including two partial differential equations and eight boundary conditions, is introduced and response of system by means of Fourier transform is obtained in the frequency domain. An exact closed-form solution is presented and frequency equation of beam is achieved. This equation is numerically solved and expressions for natural frequencies and mode shapes of beam are provided. Furthermore, the sensitivity analysis is carried out based on the variation of the right span geometrical properties. In fact, the effect of flexural rigidity and mass per unit length on the natural frequencies of the whole system is investigated and also this question is answered that adding a new span to a single-span simple beam, and increasing or decreasing the flexural rigidity and mass per unit length of it, how can affect the natural frequencies of the two-span beam?

Keywords: two-span beam, eigenvalue problem, frequency domain, natural frequencies, Euler-Bernoulli theory.

1. INTRODUCTION

Frequency analysis of beams is a classical topic in structural engineering and has attracted much attention from researchers. For instance, Rao and Mirza [1] derived the frequency and normal mode shape expressions for generally restrained Euler-Bernoulli beams with unsymmetrical translations and rotations at either end. Soon after, Lee and Ke [2] studied the problem of free vibration of a non-uniform Euler-Bernoulli beam with general elastic supports at boundary points. The integral equation and the additional algebraic equations of the vibration of beams with elastic supports and elastically mounted concentrated masses were established by Xu and Chen [3]. Li [4] used a discretization scheme based on the Galerkin method for solving the free vibration problem of beams.

Moreover, Lin and Chang [5] dealt with the free vibration analysis of a uniform multi-span Timoshenko beam with an arbitrary number of flexible constraints. Then, Lin and Tsai [6] determined the exact solutions for the natural frequencies and mode shapes of a uniform multi-span beam carrying multiple spring-mass systems. Failla and Santini [7] addressed the eigenvalue problem of Euler-Bernoulli discontinuous beams. Exact solution to free vibration of beams partially supported by an elastic foundation was given by Motaghin et al. [8].

The objective of the present study is to determine the natural frequencies and mode shapes of a two-span Euler-Bernoulli beam. The ratio of flexural rigidity and mass per unit length of the beam studied are different and variable. First, the governing differential equation of motion for Euler-Bernoulli beam is introduced. Then, the well-known Fourier transform is performed on the equation of motion and this equation is obtained in the frequency domain. It is then followed by introduction of the eigenvalue problem including a partial differential equation for each span as well as the eight pertinent boundary conditions of the problem. Next, the frequency equation of the beam is obtained, applying the boundary conditions. Furthermore, the values of the frequency parameters for different ratio of flexural rigidity and mass per unit length of the beam are given in some tables. A finite element analysis is also performed which shows the accuracy of the results. Finally discussion about the influence of the different ratio of flexural rigidity and mass per unit length of the second span on the first span concludes the paper.