



Graphical computation of gain and phase margin specifications-oriented robust PID controllers for uncertain systems with time-varying delay

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ARTICLE INFO

Article history:

Received 5 February 2010

Received in revised form 20 January 2011

Accepted 4 February 2011

Available online 10 March 2011

Keywords:

Complex Kharitonov theorem

Uncertain systems

Time-varying delay

Robust PID controller

Gain margin

Phase margin

ABSTRACT

This paper proposes a novel graphical method to compute all feasible gain and phase margin specifications-oriented robust PID controllers to stabilize uncertain control systems with time-varying delay. A virtual gain-phase margin tester compensator is incorporated to guarantee the concerned system with certain robust safety margins. The complex Kharitonov theorem is used to characterize the parametric uncertainties of the considered system and is exploited as a stability criterion for the Hurwitz property of a family of polynomials with complex coefficients varying within given intervals. The coefficients of the characteristic equation are overbounded and eight vertex Kharitonov polynomials are derived to perform stability analysis. The stability equation method and the parameter plane method are exploited to portray constant gain margin and phase margin boundaries. The feasible controllers stabilizing every one of the eight vertex polynomials are identified in the parameter plane by taking the overlapped region of the plotted boundaries. The overlapped region of the useful region of each vertex polynomial is the Kharitonov region, which represents all the feasible specifications-oriented robust PID controller gain sets. Variations of the Kharitonov region with respect to variations of the derivative gain are extensively studied. The way to select representative points from the Kharitonov region for designing robust controllers is suggested. Finally, three illustrative examples with computer simulations are provided to demonstrate the effectiveness and confirm the validity of the proposed methodology. Based on the pre-specified gain and phase margin specifications, a non-conservative Kharitonov region can be graphically identified directly in the parameter plane for designing robust PID controllers.

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1. Introduction

Uncertain systems with time-varying delay have been widely found in general process control systems, power systems, bilateral teleoperation systems, and networked control systems [1–7]. Many studies have concluded time delay will reduce the phase margin of control systems and yield reduced relative stability. Besides, system behaviors may become more complex for uncertain control systems containing time-varying delay. Thus, the presence of such time delay will greatly increase the difficulty of achieving satisfactory performance. For a review of recent results on time delay systems, see [8].

In the past years, many different controller methods have been proposed in the literature to stabilize uncertain control systems with time delay [1,9,10]. In one such work, Bozorg and Davison presented a numerical algorithm in [1] for calculating time delay stability margins in the space of time delays. According to this algorithm, the size of the stability hyperspheres space can be com-

puted. Besides, Garcia-Sanz et al. introduced two criteria based on bandwidth considerations and the quantitative feedback theory technique to ensure the robustness of the controller design when the model of the plant is not precisely known and displays variable time delay [9]. In addition, Zhao et al. integrated the pattern-based fuzzy predictive control scheme with a first-order lag unit to account for the time-delayed process dynamics with uncertainties [10]. This allows the designer to set a trade-off between performance and robustness.

On the other hand, since most industrial systems are still controlled by PID controllers, much attention has focused on stabilizing uncertain systems with or without delay using PID controllers. For instance, a computationally efficient method to characterize all stabilizing PID controllers for any given plant is provided in [11]. Extensions of the results in [11] to robustly synthesize P, PI, and PID controllers for SISO linear time-invariant plants with and without time delay exploiting the Hermite–Biehler theorem are found in [12]. Besides, in [13], the Kharitonov theorem is exploited for characterizing all PID controllers that stabilize an uncertain plant. Accordingly, a Kharitonov region in the parameter plane for scheduling the PID gains to guarantee stability and robustness is derived. Similar work to [13] is found in [14], which computes the

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