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Research paper

# Equivalent mechanical properties of biological membranes from lattice homogenization

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ABSTRACT

The goal of this manuscript is to set up a novel methodology for the calculation of the effective mechanical properties of biological membranes viewed as repetitive networks of elastic filaments, based on the discrete asymptotic homogenization method. We will show that for some lattice configurations, flexional effects due to internal structure mechanisms at the unit cell scale lead to additional flexional effects at the continuum scale, accounted for by an internal length associated to a micropolar behavior. Thereby, a systematic methodology is established, allowing the prediction of the overall mechanical properties of biological membranes for a given network topology, as closed form expressions of the geometrical and mechanical micro-parameters. The peptidoglycan and the erythrocyte have been analyzed using this methodology, and their effective moduli are calculated and recorded versus the geometrical and mechanical lattice parameters. A classification of lattices with respect to the choice of the equivalent continuum model is proposed: The Cauchy continuum and a micropolar continuum are adopted as two possible effective medium, for a given beam model. The relative ratio of the characteristic length of the micropolar continuum to the unit cell size determines the relevant choice of the equivalent medium. In most cases, the Cauchy continuum is sufficient to model membranes in most of their configurations. The peptidoglycan network may exhibit a re-entrant hexagonal lattice, for which micropolar effects become important. This is attested by the characteristic length becoming larger than the beam length for such configurations. The homogenized moduli give accurate results for both membranes, as revealed by comparison with experimental measurements or simulation results from the literature at the network scale. A first insight into the nonlinear mechanical behavior of the hexagonal and triangular networks is lastly investigated using a perturbative method.



## 1. Introduction

The membrane of biological cells is made of the assembly of filaments which are linked together as part of a network or are associated with the cell membrane to build a two-dimensional thin sheet. Two-dimensional biological networks may be wrapped around a cell as its wall or attached to its plasma or nuclear membrane. Structural elements

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