



# A constitutive model for shape memory alloys accounting for thermomechanical coupling

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## ARTICLE INFO

### Article history:

Received 13 July 2010

Received in final revised form 1 September 2010

Available online 21 September 2010

### Keywords:

Shape memory alloys

Superelasticity

Thermomechanical coupling

Loading rate effect

Finite elements

## ABSTRACT

This paper presents a generalized Zaki–Moumni (ZM) model for shape memory alloys (SMAs) [cf. Zaki, W., Moumni, Z., 2007a. A three-dimensional model of the thermomechanical behavior of shape memory alloys. *J. Mech. Phys. Solids* 55, 2455–2490 accounting for thermomechanical coupling. To this end, the expression of the Helmholtz free energy is modified in order to derive the heat equation in accordance with the principles of thermodynamics. An algorithm is proposed to implement the coupled ZM model into a finite element code, which is then used to solve a thermomechanical boundary value problem involving a superelastic SMA structure. The model is validated against experimental data available in the literature. Strain rate dependence of the mechanical pseudoelastic response is taken into account with good qualitative as well as quantitative accuracy in the case of moderate strain rates and for mechanical results in the case of high strain rates. However, only qualitative agreement is achieved for thermal results at high strain rates. It is shown that this discrepancy is mainly due to localization effects which are not taken into account in our model. Analyzing the influence of the heat sources on the material response shows that the mechanical hysteresis is mainly due to intrinsic dissipation, whereas the thermal response is governed by latent heat. In addition, the variation of the area of the hysteresis loop with respect to the strain rate is discussed. It is found that this variation is not monotonic and reaches a maximum value for a certain value of strain rate.

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## 1. Introduction

The interesting behavior of shape memory alloys (SMA) is usually attributed to their ability to undergo a reversible solid–solid phase change between a parent phase called austenite and a product phase called martensite. The transition from austenite to martensite is accompanied by a loss of crystallographic symmetry, which produces entropy and heat. Austenite can usually transform into martensite when the SMA is mechanically stressed, the resulting transformation strain can then be recovered by unloading. This seemingly elastic yet dissipative behavior is called “pseudoelasticity”.

In the last two decades, constitutive modeling of SMAs has been the topic of a large number of publications. Pan et al. (2007) developed a rate-independent model for SMAs, which was shown to fit experimental data in a number of different situations, including compression, tension–torsion and indentation experiments. Wang et al. (2008) investigated the

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