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An alternative approach to integrating plasticity relations

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ABSTRACT

A new plasticity integration algorithm is proposed based upon observations from the closed form integration of a generalized quadratic yield function over a single time step. The key to the approach is specification of the normal to the plastic flow potential as a function of the current state and strain increment. This uniquely defines the direction of the stress tensor for a convex, non-faceted flow potential. The stress magnitude and plastic strain increment are computed to satisfy the yield function. A non-quadratic, isotropic, associative flow model is coded to demonstrate accuracy and time step convergence following a step change in loading path. The model is used in additional simulations of strain localization in an expanding ring and a perforated plate.

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1. Introduction

Traditional plasticity models are defined in terms of rate equations, and the stress and any history dependent state variables must be integrated through time (Hill, 1950). In the context of a displacement based finite element code, a strain increment or an average strain rate over the time increment is provided to the plasticity algorithm, and updated stress and state variables are returned. The stability and accuracy of the material time integration algorithm may limit the time step. Consequently, the computational efficiency of simulations can be affected by the material time integration method.

Time integration algorithms for general plasticity equations typically involve subtracting a plastic strain increment from the total strain increment to arrive back at the yield surface. Extensive literature reviews of strength models and integration procedures in finite element codes are given by Yu (2002) and Kojić (2002), respectively. More recent plasticity integration algorithms have also been described (e.g. Ulz, 2009; Mosler and Bruhns, 2010; Brannon and Leelavanichkul, 2010). Details can vary considerably, but the underlying concept is common: a return mapping to the yield surface from an initial projection. Textbook descriptions (e.g. Dunne and Petrinic, 2005; Hill, 1950) often provide a 2-D depiction of the yield surface, a stress increment projecting some small distance outward from the surface, and a vector in the direction of the plastic strain increment returning to the surface, Fig. 1a. The plastic strain direction is normal to the flow potential surface. This is the same as the yield surface for associative-flow plasticity models.

The manner in which finite element analyses are run can depart significantly from this textbook illustration. Many finite element codes will take strain increments on the order of 10% or more (Abaqus, 2009). If the material yields at a strain of 0.1%, the vector projecting off the yield surface may be 100 times the radius of the surface. The plastic strain direction must project back to the surface. A slight error in the direction could result in missing the surface entirely, Fig. 1b. This projection at large strain increments is a major difficulty for integrating anisotropic material models (Kojić, 2002).

The problem is simplified considerably for J2-Flow theory and an associated flow rule. This yield surface is a hyper-sphere in deviatoric stress space. A vector passing through the center of the yield surface is colinear with the surface normal. Hence, a plastic strain increment directed toward the center of the yield surface intersects the surface at a stress consistent with the

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