



# A homogenization theory for elastic–viscoplastic materials with misaligned internal structures

T. Matsuda<sup>\*</sup>, S. Kanamaru, N. Yamamoto, Y. Fukuda

Department of Engineering Mechanics and Energy, University of Tsukuba, 1-1-1 Tennodai, Tsukuba 305-8573, Japan

## ARTICLE INFO

### Article history:

Received 16 December 2010

Received in final revised form 16 May 2011

Available online 12 June 2011

### Keywords:

Homogenization

Misalignment

Unit cell

Viscoplasticity

Plain-woven laminate

## ABSTRACT

In this study, a homogenization theory for non-linear time-dependent materials is rebuilt for periodic elastic–viscoplastic materials with misaligned internal structures, by employing a unit cell defined for the aligned structure as an analysis domain. For this, it is shown that the perturbed velocity fields in such materials possess periodicity in the directions of misaligned unit cell arrangement. This periodicity is used as a novel boundary condition for unit cell analysis to rebuild the homogenization theory. The resulting theory is able to deal with arbitrary misalignment using the same unit cell, avoiding not only geometry and mesh generation of a unit cell for every misalignment, but also the influence of mesh dependence. To verify the theory, an elastic–viscoplastic analysis of plain-woven glass fiber/epoxy laminates with misaligned internal structures is performed. It is shown that the misalignment of internal structures affects viscoplastic properties of the plain-woven laminates both macroscopically and microscopically.

© 2011 Elsevier Ltd. All rights reserved.

## 1. Introduction

The mathematical homogenization theory based on unit cell analysis (Bensoussan et al., 1978; Sanchez-Palencia, 1980; Suquet, 1987) is able to analyze both the homogenized macroscopic behavior of periodic materials, and also the microscopic stress and strain distributions in the materials. This theory therefore has been successfully applied to various analyses of mechanical properties of materials in which structural periodicity can be assumed, such as composites (e.g., Ghosh et al., 1996; Takano et al., 2000; Carvelli and Poggi, 2001; Tomita and Lu, 2002; Raghavan and Ghosh, 2005), cellular solids (e.g., Ohno et al., 2002; Okumura et al., 2004; Erami et al., 2006; Asada et al., 2009) and polycrystalline materials (e.g., Okumura et al., 2007; Watanabe et al., 2008). In this type of analysis, first, a unit cell of a periodic material is defined as an analysis domain as indicated by Y (Fig. 1a). Then, the unit cell analysis is numerically performed by imposing a periodic boundary condition on the opposite boundary edges (facets). The homogenization theory, however, encounters difficulties when dealing with materials in which misalignment of internal structures exists – see for example Fig. 1b. This is because unit cells of the sort defined for aligned cases (as shown in Fig. 1a) with periodic boundary conditions no longer work. This type of misalignment in periodic materials can become an important issue especially for laminate-structured materials, such as for example, laminate composites (Takano et al., 1995; Carvelli and Poggi, 2001), and plate-fin structures (Tsuda et al., 2010; Tsuda and Ohno, 2011). A method is therefore required which enables the homogenization theory to deal efficiently with the misalignment of internal structures.

<sup>\*</sup> Corresponding author. Tel./fax: +81 29 853 5511.

E-mail address: [matsuda@kz.tsukuba.ac.jp](mailto:matsuda@kz.tsukuba.ac.jp) (T. Matsuda).