



Application of higher-order elements in scaled boundary finite element method (SBFEM) to improve its accuracy and efficiency

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Abstract

The SBFEM is a semi-analytical fundamental-solution-less boundary-element method based only on finite elements, which combines advantages of the finite element and boundary element methods. This method has numerical solution on the boundary and analytical solution on the domain of the problem. In this paper, the Chebyshev shape functions with Clenshaw-Curtis quadrature are applied to the elements on boundary, which leads to lumped-form and blocky-lumped-form of coefficient matrices and higher-order elements. A set of wave propagation problems, subjected to various load forms are modeled using the SBFEM with very small number of degrees of freedom. The numerical results agree very well with the analytical solutions as well the results from other numerical methods.

Keywords: elastodynamics, scaled boundary finite element method, higher order element, Clenshaw-Curtis quadrature, chebyshev shape function.

1. INTRODUCTION

Many numerical methods used in elastodynamics, the most widely used computational procedure in solid mechanics are finite difference method, finite element method, boundary element method, spectral element method with their own advantages and disadvantages. In computational wave propagation, considerable efforts have been devoted for developing highly accurate numerical techniques for the solution of elastic wave equations.

Finite difference methods, which have been widely used in computational elastodynamics, require a large number of grid points to achieve the expected accuracy, even with high-order explicit or implicit spatial operators. Free surface boundaries and complex configurations coarse modeling produce lack of precision in the simulation of some problems. In practice, a difficult trade-off between numerical dispersion and computational cost is required [1].

In the finite element methods, the domain is spatially discretized. In each finite element, shape functions in the form of polynomials interpolate the displacement. Standard numerical integration of regular functions leads to the static-stiffness and mass matrices that are then assembled enforcing compatibility and equilibrium. A great flexibility exists in representing the geometry and materials, but low-order finite element methods exhibit poor dispersion properties. For the unbounded domain, the finite element method cannot represent the radiation condition at infinity exactly [2].

In the boundary-element method, only the boundary is discretized, leading to a reduction of the spatial dimension by one. The main advantages are that the solution is sought over a domain one dimension lower than the physical domain, and that the radiation is a priori satisfied. On the other hand, a fundamental solution satisfying the governing equations in the domain must be available. This analytical solution is often very complicated exhibiting singularities. Special numerical integration of the polynomials with the fundamental solution, involving singularities yields non-symmetric coefficient matrices [3].

As already mentioned, the FEM and BEM converge to the exact for decreasing element size, and special techniques and large number of elements are necessary to achieve high accuracy.