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International Journal of Pressure Vessels and Piping

journal homepage: www.elsevier.com/locate/ijpvp

Short Communication

# Linear and non-linear analyses for semi-elliptical surface cracks in pipes under bending

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#### ARTICLE INFO

Article history: Received 26 May 2009 Received in revised form 1 June 2010 Accepted 8 November 2010

Keywords: Fracture Finite element method Shape factor function Semi-elliptical surface cracks J integral Reference stress approach

#### ABSTRACT

In this paper, the *J* integral was calculated for semi-elliptical surface cracks in pipes under bending using three-dimensional finite element analysis. The computations were performed for elastic and elastic-plastic behaviours. For the elastic case, the numerical results allowed the extrapolation of shape functions for analytical determination of the *J* integral. The results are in a good agreement with those in the literature if the ratio between the radius and the thickness of the pipe (*R*/*t*) is from 1 to 10. The analysis was extended to values of the ratio *R*/*t* higher than 10. For the elastic-plastic, the numerical results are in good agreement with the analytical solution found in the literature for thick pipes (*R*/*t* ≥ 10). The effect of the ratio *R*/*t* becomes sensible when the ratio of the applied moment to the moment of reference (*M*/More) exceeds 0.9.

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## 1. Introduction

External cracks can occur in many structural components of cylindrical form. They are the cause of premature damage in structures such as piping, bolts, pins and reinforcements of aircraft.

The behaviour of external cracks in pipes has been the focus of many researches in the past. Marie et al. [1] presented numerical solutions for external cracks in plates, pipes and elbows.

Folias [2,3] obtained solutions of the stress intensity factor for a circumferential crack in a pipe subjected to internal pressure and tensile loading, using classical shell theory (without consideration of transverse shear strain). The equations obtained by Folias are valid only for short cracks, Duncan-Fama and Sanders [4] extended the solutions to longer cracks using a numerical technique. Afterwards, a semi-analytical approach based on complete shell theory including transverse strain was developed by Sanders [5,6] to derive the solutions for a circumferential crack in a pipe subjected to axial load and bending moment. Later, Klecker et al. [7] extended these solutions to include the effect of *R/t* when this ratio varies between 5 and 10. From the work of Sanders, Zahoor [8] developed another set of closed form expressions and these expressions were included in the Ductile Fracture Handbook published later. Delale

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0308-0161/\$ – see front matter  $\odot$  2010 Elsevier Ltd. All rights reserved. doi:10.1016/j.ijpvp.2010.11.001

and Erdogan [9] and German et al. [10] obtained stress intensity factors for interior and exterior circumferential surface cracks using the line-spring model. Kim et al. [11] analyzed non-linear fracture mechanics parameters for pipes with part circumferential inner surface cracks, subject to internal pressure and global bending. They proposed solutions in the form of the reference stress approach. The majority of the studies are limited to the case of R/t < 20 for elastic-plastic behaviour.

In this study, the behaviour of semi-elliptical surface cracks in pipes under bending was analyzed using the finite element method. The J integral around the crack tip was computed for elastic and elastic-plastic behaviour of the materials. Analytical equations were obtained for calculating J [12].

## 2. Finite element analysis

### 2.1. Geometrical model

This study presents a three-dimensional finite element analyses by the code ABAQUS for semi-elliptical surface cracks in pipes. The pipes were subjected to bending load. The ratio of crack depth to crack length (a/c) ranged from 0.6 to 0.8, the ratio of crack depth to wall thickness (a/t) ranged from 0.2 to 0.8, and the ratio of the internal radius to the wall thickness (R/t) varied from 1 to 80. Fig. 1 presents the geometrical model used in the study.