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Non-cyclic shakedown/ratcheting boundary determination – Part 1: Analytical approach

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ABSTRACT

From the practical point of view, the classical elastic shakedown methods are not very useful for design, since in most components the stresses can safely exceed the elastic limit locally. This paper generalizes the static shakedown theorem (Melan's theorem) to allow the analysis of plastic shakedown. Since the method is derived from a lower bound formulation in shakedown, it is very useful for the design purposes (safe). The ratchet boundary is analytically determined using the proposed method for several examples with uniform stress distributions. The numerical implementation of the method along with several examples is discussed in an accompanying paper.

1. Introduction

Components subjected to a combination of cyclic and steady loads can have two different asymptotic deformation responses, a strictly repetitive cyclic one (shakedown) or a continual dimensional change with each cycle (ratcheting or incremental collapse). For the design of components, ratcheting is usually not an acceptable behaviour.

In predicting whether shakedown or ratcheting will occur, the asymptotic behaviour of the component under a given system of loads must be determined. One way to do this is to directly simulate the cyclic load application over a number of cycles until a repetitive response is observed. In general, this requires a cyclic elastic-plastic analysis. The other way is to obtain the asymptotic solution directly, usually through the use of the classical shakedown theorems. Although there is no known theoretical reason that one of these methods should be generically more efficient than the other, it seems that in practice the direct methods are often the more efficient ones. However, direct methods are harder to implement in a universally valid way, and the use of the classical shakedown theory is additionally limited to cases where the asymptotic response of the structure is purely elastic.

The classical shakedown theory consists of Melan's theorem [1] and of Koiter's theorem [2]. Melan's [1] theorem derives from equilibrium formulations, giving rise to a lower bound estimate of the shakedown load. Koiter's [2] theorem is based on compatibility considerations and estimates an upper bound shakedown load. These bounding theorems assume elastic-perfectly plastic material behaviour and neglect the effect of any geometry changes in the component. From the practical point of view, the classical shakedown concepts are rarely useful as a design limit because most components contain stress raisers or local discontinuities such as notches, and the magnitude of the stresses may locally exceed the elastic range of the material without causing integrity concerns. In other words, in many practical cases, the classical shakedown theorems are not able to predict the relevant boundary between shakedown and ratcheting (ratchet boundary). Therefore, an alternative method to predict the loads at the onset of ratcheting is desirable.

Unlike for elastic shakedown, no classical approach exists to assess plastic shakedown (shakedown to alternating plasticity). However, several researchers have proposed computational methods for plastic shakedown analysis.

Kalnins [3] proposed a method based on the elastic core concept. The principle behind this method is that by having a continuous elastic core in a structure, an incremental collapse cannot occur under load cycles. Particularly in cases where local plasticity at discontinuities dominates, this method requires only a small number of elastic-plastic cycles to achieve a sufficiently stable elastic core; however, in general the computational effort may still be significant.

Chen and Ponter [4] proposed a computational process for identifying the shakedown-ratchet boundary based on an

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