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Analysis of residual stresses in a long hollow cylinder

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ABSTRACT

This paper presents an analytical method for solving the axisymmetric stress problem for a long hollow cylinder subjected to locally-distributed residual (incompatible) strains. This method is based on direct integration of the equilibrium and compatibility equations, which thereby have been reduced to the set of two governing equations for two key functions with corresponding boundary and integral conditions. The governing equations were solved by making use of the Fourier integral transformation. Application of the method is illustrated with an analysis of the welding residual stresses in a butt-welded thick-walled pipe.

1. Introduction

Welding residual strains substantially affect the lifetime of welded structural members [1]. The locally-distributed fields of technological residual strains occur in welded joints due to non-uniform heating and cooling during the welding and aftercooling processes as a result of phase change in the zones of thermal influence ([2], [3]: pp 75–87). Under certain conditions, the residual stresses cause brittle failure of long-term-functioning welded structures. Since an analysis of the residual stress-state induced by the welding residual strains is very important for a proper inspection of welded joints, it presents a vital issue for specialists in both academia and industry.

There are different approaches for analysis of the residual stresses [3,4] in butt-welded joints, which mostly combine both experimental [5–8] and theoretical [9,10] techniques. A vast majority of the existing experiment-calculated methods employ numerical [11–18] or approximate [19,20] procedures. However, for efficient analysis of the residual stresses, as well as for solving inverse elasto-plastic problems (which necessarily occur when one applies certain experiment-calculated methods), exact analytical solutions are required.

One of the most efficient theoretical approaches to determination of the residual stresses is based on the method of conventional plastic strains [19,21]. According to this method, the material of a welded solid is assumed to be elastic at some distance from the axes of welded joints. But within the zones of thermal influence due to welding, the material is assumed to be elastic-plastic. Consequently, the components of the strain-tensor can be presented in the form

$$e_i = \tilde{e}_i + \epsilon_i, \quad e_{ij} = \tilde{e}_{ij} + \gamma_{ij}, \ \ ij = ji, \ \ i \neq j, \tag{1}$$

where the indexes i and j show the coordinate directions in the chosen coordinate system; e_i and e_{ij} are the normal and shear strains-tensor components, respectively; $\tilde{e}_i$ and $\tilde{e}_{ij}$ denote the elastic strains; ϵ_i and γ_{ij} stand for the residual strains. The residual strains are locally-distributed in the neighborhood of a welded joint and vanish at a distance from it. In more general form, the representation of total strains has been given in [9,22]. The method of conventional plastic strains has been sufficiently employed for analysis of the residual stresses in welded structure members of various shape [23–25].

Among structural elements of different shape, cylindrical bodies are widely used in engineering practice as elements of pipelines, pylons, crosstops, pressure vessels, etc. In most cases, the residual stresses in butt-welded hollow cylinders can be analyzed with application of an axisymmetric model. By making use of the aforementioned method of conventional plastic strains, the problem for determination of the residual stresses in a butt-welded cylindrical pipe can be reduced to the axisymmetric elasto-plastic problem and then solved by means of the methods of elasticity.

There exist a great number of methods for determination of the stresses and displacements in a long hollow cylinder subjected to various types of external loading. A great many of the dominant methods are based on application of harmonic or biharmonic stress

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