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A **Innovative Efficient Element for Analysis of Axially FGM Tapered Beams Using FEM**

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Abstract

This paper aims at presenting a new efficient element for free vibration of Axially Functionally Graded Materials (FGMs) non-prismatic beams using Finite Element Method (FEM). Using concept of Basic Displacement Functions (BDFs), two- node element extends to three-node element for obtaining much more exact results using FEM. First, BDFs are introduced and computed using energy method such as unit-dummy load method. Afterward, new shape functions are developed in terms of BDFs during the procedure based on the mechanical behavior of the element in which presented shape functions benefit generality and accuracy from stiffness and force method, respectively. Finally, deriving structural matrices of the beam with respect to new shape functions; free vibration of the FGM beam is studied using finite element method for all types of AFGM beams and the convergence of FEM has been studied. The results from free vibration is in perfect agreement with those of previously published.

Keywords: Axially Functionally Graded Materials (AFGM), Finite Element Method (FEM), Basic Displacement Functions (BDFs), Free Vibration.

1. Introduction

Functionally graded materials (FGMs) are a new class of advanced composite materials possessing continuous variation of material properties with respect to the spatial coordinates. Unlike laminated composites, which are prone to interfacial stress concentration, leading to delamination and propagation of cracks, FGMs exhibit smooth and gradual vary in material properties. This can be achieved by either continuous change in thickness direction or smooth change of in-plane materials. Most of researchers have focused on the FG beams while material properties fluctuate along the dimensions of cross-section all together or independently for beam or plate [1-4].However, a few of these studies deal with FGMs with materials indices varying through the beam's length [5-27].

The majority of researches conducted in this field are concerned with presenting closed-form solutions. For example, Elishakoff *et al*.[5-18] have applied the semi inverse method in which the closed-form solution of the problem is applicable to the particular problems; the semi-inverse method is only useable for the beams with specific required displacement and a physical property such as mass density. Moreover, other physical property which mainly is the modulus of elasticity, is derived through the satisfying the governing differential equation of the problem. Huang and Li [19] have studied free vibration of axially FG beam with non-uniform cross-sectional area for various flexural rigidity and mass density cases by transforming differential equation into Fredholm integral equation. Alsharbagy *et al*.[20] adopted FEM to investigate the dynamic characteristics of axially FG beam. Singh *et al*.[21] also probed stability of non-uniform axially FG beam through modeling non-prismatic beams as an assemblage of several uniform segments. Furthermore, Shahba & Rajasekaran [22] employed two different numerical methods to investigate the free vibration and instability analysis of axially functionally graded beam.

Attarnejad*et al*. [23-26] have analyzed axially FG Euler-Bernoulli and Timoshenko beam using FEM in which new functions, namely Basic Displacement Functions (BDFs) have been introduced. First, a two-node element has been considered. Then, Basic Displacement Functions (BDFs) are defined and derived for this element considering each node. Afterwards, new shape functions are expressed in terms of BDFs and obtained from a mechanical point of view. Finally, structural stiffness and consistent mass matrix are derived and presented static and free vibration analysis of the FG beams using finite element method. In addition,