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Numerical solution of Burger’s equation via RBF-collocation scheme

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Abstract

A classical problem in computational hydraulics and/or hydrology is to quantify the spatial as well as temporal distribution of pressure and velocity fields within a water body. This quantification plays an important role in analysis and design of hydraulic structures, irrigation and drainage networks, and water transmission lines. In this regard, numerous numerical techniques have been devised to analyze the governing equations (e.g., Navier-Stokes equations) and its variants such as Burger’s equation with new scheme usually emerging from new technology. In this study, RBF-collocation scheme is implemented to solve the one dimensional nonlinear Burger’s equation. This particular equation is chosen to better assess and evaluate the performance of the cited numerical scheme as the equation possesses analytical exact solution under certain conditions. Conversion of partial differential equation into system of nonlinear algebraic equations is achieved by spatial approximation of the governing equation via RBF-collocation and temporal approximation via finite difference schemes. In contrast to the current research on RBF-collocation where Radial Basis Functions are considered to capture the spatial variation and RBF coefficients are introduced to mimic the temporal variation resulting in system of nonlinear ODEs in terms of coefficients, a special form of RBF-collocation is proposed to keep the original state variable(s) intact. The new proposal was found to be efficient and robust. Performance assessment is achieved by comparing the numerical solution with analytical solution available in the relevant literature. Results show that the RBF-collocation scheme is quite stable and accurate with two main advantages. First, the structures of RBF shape functions allow appropriate selection of nodal distribution consistent with process physics. Secondly, RBF-collocation is quite capable of solving the governing partial differential equations using a few grid points. In conclusion, RBF-collocation is considered to be an efficient scheme for obtaining numerical solution of Burger’s equation.

Keywords: RBF-collocation, shape parameter, Burger's equation

1- INTRODUCTION

Simulation of unsteady flow in open channels has always been of great interest to hydraulic engineers. Transfer of information (i.e., water depth and discharge), design of flood warning systems and reservoir operation are typical examples of outcome emerging from such simulation. Acknowledging the fact that the flood wave is usually unsteady in practice, simulation of unsteady flow plays an important role in analysis and design of hydraulic structures.

Simulation of transient flow in open channels can be achieved in three different ways, namely; hydraulic, hydrologic and data-driven approaches. In hydraulic routing invariably called distributed flow routing or white box approach, conservation of mass and momentum in differential form will be utilized to simulate water flow in waterways. In hydrologic routing invariably called lumped flow routing or grey box approach, the conservation of mass in lumped format coupled with empirical storage function will be implemented to transfer the required information from one section to another. In data-driven approaches invariably called empirical flow routing or black-box approach, no accommodation for conservation laws will be provided. Extensive input-output data are required to train the transfer function model under consideration. As the routing methods move from more physically-based to a more empirically based, the degree of input-output data requirement will be increased dramatically.

There are numerous benefits in using the hydraulic flow routing the most important of which is its distributed nature. The distributed nature would help to map flood and delineate floodplain in natural rivers. The governing equations in hydraulic flow routing are Saint-Venant equations. Saint-Venant equations are a