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# Kinematic FE limit analysis homogenization model for masonry walls reinforced with continuous FRP grids

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#### ABSTRACT

A homogenization model for periodic masonry structures reinforced with continuous FRP grids is presented. Starting from the observation that a continuous grid preserves the periodicity of the internal masonry layer, rigid-plastic homogenization is applied directly on a multi-layer heterogeneous representative element of volume (REV) constituted by bricks, finite thickness mortar joints and external FRP grids. In particular, reinforced masonry homogenized failure surfaces are obtained by means of a compatible identification procedure, where each brick is supposed interacting with its six neighbors by means of finite thickness mortar joints and the FRP grid is applied on the external surfaces of the REV. In the framework of the kinematic theorem of limit analysis, a simple constrained minimization problem is obtained on the unit cell, suitable to estimate – with a very limited computational effort – reinforced masonry homogenized failure surfaces.

A FE strategy is adopted at a cell level, modeling joints and bricks with six-noded wedge shaped elements and the FRP grid through rigid infinitely resistant truss elements connected node by node with bricks and mortar. A possible jump of velocities is assumed at the interfaces between contiguous wedge and truss elements, where plastic dissipation occurs. For mortar and bricks interfaces, a frictional behavior with possible limited tensile and compressive strength is assumed, whereas for FRP bars some formulas available in the literature are adopted to reproduce the delamination of the truss from the support.

Two meaningful structural examples are considered to show the capabilities of the procedure proposed, namely a reinforced masonry deep beam ( $0^{\circ}/90^{\circ}$  continuous reinforcement) and a masonry beam in simple flexion for which experimental data are available. Good agreement is found between present model and alternative numerical approaches.

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### 1. Introduction

The recent earthquake occurred in Abruzzo (Italy 2009) indicated once again that the historical buildings, largely constituted by masonry structures, are scarcely resistant to horizontal loads and highly vulnerable to seismic actions. Such inadequate behavior of brickwork under earthquakes is a common issue of many masonry buildings and is due to several concurring factors, among the others the most important being the low strength of mortar joints, which represent preferential planes of weakness where cracks propagate.

Conventional retrofitting techniques, such as external reinforcement with steel plates, surface concrete coating and welded mesh, have proven to be impractical, time expensive and add mass to the structure (which may increase earthquake-induced inertia forces). In this context, the utilization of G-CFRP grids as reinforcement (see Fig. 1) instead of conventional methods seems the most suitable

\* Tel.: +39 022399 4225; fax: +39 022399 4220. *E-mail address:* milani@stru.polimi.it solution (Corradi et al., 2002; Eshani, 1997; Korany and Drysdale, 2007; Triantafillou, 1998; Papanicolaou et al., 2007, 2008), for their limited invasiveness and good performance at failure.

Several research efforts have been focusing on the utilization of glass instead of carbon in the realization of the composite fibers, due to the remarkable differences in the production cost. As well known, GFRP mechanical performance is substantially lower to CFRP, requiring to use much more strips (thus enlarging the reinforced area) if glass is used instead of carbon (CNR, 2004; Focacci, 2008). Due to both the constant reductions of the installation costs of FRP and the requirement to reinforce more heavily, the utilization of continuous FRP grids is becoming popular and, in some cases, such new technology is preferred to the more traditional utilization of strips and cables.

Unfortunately, while the practical usage of such technology is spreading, no numerical models are available in the literature to predict the mechanical behavior of such kind of reinforcement near the collapse and a few information is available for design purposes on codes of practice (CNR, 2004; JSCE, 2001).

From a theoretical point of view, the complexity of the problem, which involves fragile phenomena in both masonry and the bond

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