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Analytical investigation for true and spurious eigensolutions of multiply-connected membranes containing elliptical boundaries using the dual BIEM

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ABSTRACT

It is well known that the boundary element method may induce spurious eigenvalues while solving eigenvalue problems. The finding that spurious eigenvalues depend on the geometry of inner boundary and the approach utilized has been revealed analytically and numerically in the literature. However, all the related efforts were focused on eigenproblems involving circular boundaries. On the other hand, the extension to elliptical boundaries seems not straightforward and lacks of attention. Accordingly, this paper performs an analytical investigation of spurious eigenvalues for a confocal elliptical membrane using boundary integral equation methods (BIEM) in conjunction with separable kernels and eigenfunction expansion. To analytically study this eigenproblem, the elliptic coordinates and Mathieu functions are adopted. The fundamental solution is expanded into the separable kernel by using the elliptic coordinates and the boundary densities are expanded by using the eigenfunction expansion. The Jacobian terms may exist in the separable kernel, boundary density and boundary contour integration and they can cancel each other out. Therefore, the orthogonal relations are reserved in the boundary contour integration. In this way, a similar finding about the mechanism of spurious eigenvalues is found and agrees with those corresponding to the annular case. To verify this finding, the boundary element method and the commercial finite-element code ABAQUS are also utilized to provide eigensolutions, respectively, for comparisons. Good agreement is observed from comparisons. Based on the adaptive observer system, the present approach can deal with eigenproblems containing circular and elliptical boundaries at the same time in a semi-analytical manner. By using the BIEM, it is found that spurious eigenvalues are the zeros of the modified Mathieu functions which depend on the inner elliptical boundary and the integral formulation. Finally, several methods including the CHIEF method, the SVD updating technique and the Burton & Miller method are employed to filter out the spurious eigenvalues, respectively. In addition, the efficiency of the CHIEF method is better than those of the SVD updating technique and the Burton & Miller approach, since not only hypersingularity is avoided but also computation effort is saved.

1. Introduction

Eigenanalysis is very important for vibration and acoustics, because it can provide some fundamental information. Since analytical solutions are sometimes not available, numerical methods are needed. In recent years, several numerical methods were utilized to determine eigenvalues and eigenmodes such as the finite element method (FEM) or the boundary element method (BEM). Although the FEM is a popular method, it needs to generate the mesh over the whole domain. The BEM only generates the mesh on the boundary but it may face with the calculation of the principal value and the pollution of spurious eigenvalues. Tai and Shaw (1974) first employed the complex-valued BEM to solve membrane vibration. De Mey (1976) revisited this problem in 1976. Later, De Mey (1977) proposed a simplified approach by using only the real-part or imaginary-part kernel where he found that spurious solutions were imbedded as well as the ill-posed matrix appeared. In a similar way of using the real-part kernel, Hutchinson and Wong (1979) and Hutchinson (1984) solved the free vibration of plate. Also, Yasko (2000) as well as Duran et al. (2001) employed the real-part kernel approach. It is interesting to find that Kang et al. (1999) proposed a non-dimensional influence function (NDIF) method which was an imaginary-part kernel approach as

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