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An augmented multicrack elastoplastic damage model for tensile cracking

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ABSTRACT

In this paper we present the general formulation and numerical aspects of an augmented multicrack elastoplastic damage model aiming to reflect the crack induced anisotropy in concrete like quasi-brittle materials. Consistent evolution laws for the involved internal variables are derived based on the augmented Lagrangian method. The (time) discrete formulation and the corresponding variational structure are investigated, with the Euler–Lagrangian equations defining the closest-point projection approximation of the proposed model. The numerical aspects, such as the stress updating algorithm and the algorithmic consistent tangent moduli, are also discussed in details. It is found that in the developed numerical algorithm the active loading surfaces are determined in such a posterior manner that potential numerical problems due to the iteratively updating procedure in classical algorithms can be avoided. The proposed model is applied to the modeling of tensile cracking in concrete. The behavior of a single crack is characterized by an elliptical cracking surface and a hyperbolic softening function, with the orientations of potential cracks determined by Mohr's postulate. The model is verified by calculating the single point stress vs. strain relations of concrete under several typical proportional and non-proportional loading cases. Finally, two benchmark tests of concrete structures, i.e. four-point bending beam under cyclic loading (Hordijk, 1992) and double edge notched specimens under mixed tension/shear forces (Nooru-Mohamed, 1992), are numerically simulated. Both predicted load vs. displacement curves and crack patterns agree well with the experimental data.

1. Introduction

Stiffness degradation due to microcracks/microvoids evolution and irreversible deformations caused by plastic slips are the main mechanisms responsible for the nonlinear behavior of both quasi-brittle geotechnical materials and ductile metals. Taking the tensile cracking in plain concrete as a typical example, localized microcracks generally form in the interfacial transition zones between cement pastes and coarse aggregates. Macroscopically, the material exhibits a stress vs. strain (or displacement) curve with a softening regime right after the stress apex. The fact that these microcracks mainly evolve "in the direction perpendicular to the maximum tensile strain or stress" (Krajcinovic and Fonseka, 1981) inevitably induces an anisotropic stiffness degradation in the initially isotropic material, i.e. the so-called damage or crack induced anisotropy. Moreover, irreversible deformations caused by the frictional slips along some preferred crack lips and by the incomplete closure of cracks upon unloading are also experimentally observed. The realistic consideration of above inelastic behavior is of great significance to the numerical modeling of tensile cracking in concrete.

Thereby, it is not surprising that plasticity and damage mechanics have been widely adopted in the constitutive modeling of engineering materials; see Chen (1994) and Krajcinovic (2003) and the references therein. The concepts of plasticity and damage mechanics are rather similar and share some common features, e.g. a yield or damage loading surface and an evolution law for the plastic or damage internal variable. The similarities and differences between the plastic model (Chen, 1994) and the damage (elastic degradation) theory (Dougill, 1976; Hueckel and Maier, 1977; Ortiz, 1985; Simo and Ju, 1987; Ju, 1989; Yazdani and Schreyer, 1990; Simo et al., 1993; Lubarda et al., 1994) were discussed in details by Carol et al. (1994).

To reflect the nonlinear behavior of many engineering materials, a single smooth loading surface is in general insufficient. Accordingly, on the one hand, models with non-smooth surface composed of several smooth functions have been proposed, such as Tresca model for metal, Cam–Clay model for soil, Mohr–Coulomb model for rock (Chen, 1994), models with combined Rankine and Drucker–Prager surface for concrete (Feenstra and de Borst, 1995; Meschke et al., 1998), and so on. On the other hand, multiple yield or damage criterion have also been adopted in constitutive models, e.g. "slip theory of plasticity" for crystals (Batdorf and

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