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Evolution of yield surface in the 2D and 3D stress spaces

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ABSTRACT

Initial and subsequent yield surfaces for 6061 aluminum, determined by a method of automated yield stress probing, are presented in the 2D ($\sigma_{zz}-\sigma_{\theta z}$) and 3D ($\sigma_{\theta\theta}-\sigma_{zz}-\sigma_{\theta z}$) stress spaces. In the ($\sigma_{zz}-\sigma_{\theta z}$) space, yield surfaces at small pre-strains show the noses and unapparent cross effect. At larger pre-strains, they become ellipses with positive cross effect. In the ($\sigma_{\theta\theta}-\sigma_{zz}-\sigma_{\theta z}$) space, the initial yield surface is not well described by von Mises yield criterion due to material anisotropy. The yield surfaces of various torsional pre-strains show obvious rotation around the σ_{zz} axis but they do not rotate when subjected to axial pre-strains. Therefore, the rotation behavior of yield surface is pre-strain path dependent. The rotation of yield surfaces in the 3D space is the emphasis of the present paper. Coupled axial–torsional behavior subjected to torsion after axial pre-strain are also presented for the same material that is used to determine the yield surfaces. This information is useful for verification of constitutive models.

1. Introduction

Evolution of yield surface is one of important characteristics of plastic behavior. Three main modes of yield surface evolution, including isotropic expansion or contraction, translation, and distortion, are well recognized by researchers. However, the rotation of yield surface has not been much investigated. The yield surfaces are influenced by many factors. The most obvious one is the difference in material, including different heat treatments. In addition, the initial anisotropy induced by the manufacturing process is quite important. The anisotropic materials show more complicated behavior under plastic deformation by inducing additional anisotropy. For small plastic pre-strains of less than 1%, experimental results showed no cross effect (Phillips and Tang, 1972; Phillips et al., 1974; Phillips and Moon, 1977; Moreton et al., 1978; Wu and Yeh, 1991). For plastic pre-strains larger than 1%, two different kinds of evolution of yield surfaces including expanding with positive cross effect and shrinking with negative cross effect were observed (Hecker, 1971; Helling et al., 1986; Khan et al., 2009, 2010a,b; Ng et al., 1979; Shiratori et al., 1973; Stout et al., 1985; Wu, 2003). There were also experimental results which showed contraction first and then expansion with increasing plastic pre-strains (Helling et al., 1986; Shiratori et al., 1973; Williams and Svensson, 1971). More information about the evolution of yield surface was reported in Ellis et al. (1983), Boucher et al. (1995), Lissenden and Lei (2004), and in the book (Wu, 2005).

The experimental study of the yield surface was generally conducted on plane stress specimens including plate-like specimens or thin-walled tubes. For rolled plates or cross-shaped specimens, tensile tests with different loading axes or biaxial testing were mostly used (Ikegami, 1975a,b; Kreissig and Schindler, 1986; Losilla and Tourabi, 2004). In thin-walled tubes with z denoting the axial direction and θ the circumferential direction, the traditional axial–torsional testing was limited to yield loci of the ($\sigma_{zz}-\sigma_{\theta z}$) space with zero hoop stress $\sigma_{\theta\theta}$, where σ_{zz} was the axial stress and $\sigma_{\theta z}$ was the shear stress. Additional internal or external pressure was needed to obtain stress states in the ($\sigma_{\theta\theta}-\sigma_{zz}$) space or in the ($\sigma_{\theta\theta}-\sigma_{zz}-\sigma_{\theta z}$) space. Probing subsequent yield surfaces in the half ($\sigma_{\theta\theta}-\sigma_{zz}$) space, only tension in the hoop stress, can be accomplished by simply applying axial load and internal pressure. These works were presented in Lipkin and Swearengen (1975), Phillips and Das (1985), Khan et al. (2010b). Applying additional external pressure was harder but needed to probe yield surfaces in the whole ($\sigma_{\theta\theta}-\sigma_{zz}$) space, including compression in hoop stress, and the works were reported by Shiratori et al. (1973) and Moreton et al. (1978). There have been only a few experimental results devoted to the determination of yield surface in the ($\sigma_{\theta\theta}-\sigma_{zz}-\sigma_{\theta z}$) space. Shiratori's group (Shiratori et al., 1973) investigated subsequent yield surfaces in the ($\sigma_{\theta\theta}-\sigma_{zz}-\sigma_{\theta z}$) space by plotting contour lines representing constant axial stresses or shear stresses in the ($\sigma_{zz}-\sigma_{\theta z}$) or ($\sigma_{\theta\theta}-\sigma_{zz}$) space, respectively. However, data points in that paper were insufficient to draw contour lines in detail. Phillips and Das (1985) determined initial and subsequent yield surfaces in the ($\sigma_{\theta\theta}-\sigma_{zz}-\sigma_{\theta z}$) space and presented yield loci of the yield ellipsoid cut by various planes parallel to the $\sigma_{\theta z}$ axis.

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