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A Bending-Gradient model for thick plates, Part II: Closed-form solutions for cylindrical bending of laminates

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ABSTRACT

In the first part (Lebée and Sab, 2010a) of this two-part paper we have presented a new plate theory for out-of-plane loaded thick plates where the static unknowns are those of the Kirchhoff–Love theory (3 in-plane stresses and 3 bending moments), to which six components are added representing the gradient of the bending moment. The new theory, called Bending-Gradient plate theory is an extension to arbitrarily layered plates of the Reissner–Mindlin plate theory which appears as a special case when the plate is homogeneous. Moreover, we demonstrated that, in the general case, the Bending-Gradient model cannot be reduced to a Reissner–Mindlin model. In this paper, the Bending-Gradient theory is applied to laminated plates and its predictions are compared to those of Reissner–Mindlin theory and to full 3D (Pagano, 1969) exact solutions. The main conclusion is that the Bending-Gradient gives good predictions of deflection, shear stress distributions and in-plane displacement distributions in any material configuration. Moreover, under some symmetry conditions, the Bending-Gradient model coincides with the second-order approximation of the exact solution as the slenderness ratio L/h goes to infinity.

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1. Introduction

Laminated plates are widely used in engineering applications. For instance angle-ply carbon fiber reinforced laminates are commonly used in aeronautics. However, these materials are strongly anisotropic and the plate overall behavior is difficult to capture. The most common plate theory is the Kirchhoff–Love plate model. However, it is well-known that, when the plate slenderness ratio L/h (h is the plate thickness and L the span) is not large enough, transverse shear stresses which are not taken into account in the Kirchhoff–Love theory have an increasing influence on the plate deflection.

In recent decades many suggestions have been made to improve the estimation of transverse shear stresses. Reddy (1989), Noor and Malik (2000), and Carrera (2002) provided detailed reviews for these models. Two main approaches can be found: asymptotic approaches and axiomatic approaches. The first one is mainly based on asymptotic expansions in the small parameter h/L (Caillerie, 1984; Lewinski, 1991a,b,c). However, higher-order terms yield only intricated "Kirchhoff–Love" plate equations and no distinction between relevant fields and unknowns was made. The second main approach is based on assuming *ad hoc* displacement or stress 3D fields. These models can be "Equivalent Single Layer" or "Layerwise". Equivalent Single Layer models treat the whole laminate as an equivalent homogeneous plate. However, when dealing with laminated plates, these models lead systematically to discontinuous transverse shear stress distributions through the thickness as indicated by Reddy (1989). In layerwise models, all plate degrees of freedom are introduced in each layer of the laminate and continuity conditions are enforced between layers. The reader can refer to Reddy (1989) and Carrera (2002) for detailed reviews of kinematic approaches and to Naciri et al. (1998), Diaz Diaz et al. (2001, 2007), Hadj-Ahmed et al. (2001), Caron et al. (2006), and Dallot and Sab (2008) for static approaches. Layerwise models lead to correct estimates of local 3D fields. However, their main drawback is that they involve a number of degrees of freedom proportional to the number of layers. The limitation is immediately pointed out with functionally graded materials, where the plate constituents properties vary continuously through the thickness (Nguyen et al., 2008a,b).

In the first part of this work (Lebée and Sab, 2010a) we revisited the use of 3D equilibrium in order to derive transverse shear stress as Reissner (1945) did for homogeneous plates. Thanks to standard variational tools, this led us to an Equivalent Single Layer plate theory which takes accurately into account shear effects and does not require any specific constitutive material symmetry: the Bending-Gradient theory. This plate theory is identical to the Reissner–Mindlin plate theory in the case of homogeneous plates. However, for laminated plates, shear forces are replaced by the gradient of the bending moment $\mathbf{R} = \mathbf{M} \otimes \boldsymbol{\nabla}$. Hence, this theory belongs to the family of higher-order gradient models. The mechanical

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