



Analysis of Knoop indentation

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ABSTRACT

Knoop indentation tests have long been a standard method for material characterization due to the fact that they provide an easy, inexpensive non destructive and objective method of evaluating basic properties from small volumes of materials. In spite of the broad use, Knoop indentation has never been analysed and its methodology is basically empirical. The present work presents an extensive finite element study on the adhesionless contact of flat surfaces by Knoop indenter. The aim of this work is to explore the theoretical foundation for the commonly used Knoop test and shed light to the interesting details that make the Knoop test so useful and simple. Both elastic and elastoplastic responses are explored. The material of the contacting solid is modeled as homogeneous and isotropic. The effect of the Coulomb friction at the contact region is also considered. Subsequently, the computational results presented in the current study are compared with analytical and experimental results that exist in bibliography.

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1. Introduction

Indentation tests have long been a standard method for material characterization, [Tabor \(1951\)](#) and [Mott \(1956\)](#). When testing materials in layered components of micro-electro-mechanical devices, polymer films, paintings, biomaterials, etc. the volumes under investigation are very small while the environmental conditions can make classical mechanical tests difficult to perform. Such conditions may arise when materials operate at cryogenic temperatures, in devices like superconducting electric circuits, liquid hydrogen bubble chambers and cryogenic fuelled rockets or at low temperatures that reduce phenomena such as thermal vibration and diffusion and allow fundamental studies on dislocation movements. In such conditions micro-indentations are good alternative tests that can be adopted to perform in harsh environments, however they require accurate analysis.

In recent years, instrumented indentation, using preferably pyramid indenters such as Vickers, Berkovich and Knoop, proved to be very useful in testing small material volumes, [Fischer-Cripps \(2002\)](#). Although instrumented indentation has been in use for more than 20 years, fundamental issues remain to be cleared. Such issues include the systematic investigation of the elastic and elastoplastic indentation by pyramid indenters. Standard geometries for pyramid indenters are three types of pyramids under the well established terms of [Smith and Sutherland \(1925\)](#), [Berkovich \(1951\)](#) and [Knoop et al. \(1939\)](#). Their shapes are normal pyramids with square, regular triangle and rhombus bases, respectively.

Their tips are unavoidably slightly rounded; however, the influence of roundness is not always of major concern, especially when indentation is sufficiently deep. The main obstacle for such investigations is the difficulty in the non-linearity of the contact analysis and the computational demands that present the three-dimensional aspects of the problem.

Numerical work on the subject has been attempted in recent years. The Vickers and Berkovich indentation methods have been examined by [Giannakopoulos et al. \(1994\)](#), [Murakami et al. \(1994\)](#) and [Larsson et al. \(1996\)](#), respectively. In these works the finite element method (FEM) was incorporated in three-dimensional models in order to explore the theoretical foundation for the commonly used tests while the response for isotropic elasticity and large elastoplastic deformations was examined. The theoretical findings were compared with experimental results, performed both on the nano- and microscale.

On the other hand, the Knoop indentation has not received the appropriate attention in terms of theoretical investigation. Because Knoop indentation is less symmetric than Vickers and Berkovich indentation, its numerical analysis is harder to perform. The only numerical work on Knoop indentation that we are aware of was presented by [Rabinovich and Savin \(1996\)](#) where a new methodology for solution of the three-dimensional contact based on the variational and boundary integral approach was developed, in the context of linear elasticity. Rabinovich and Savin presented approximately the shape of the contact pressure field as well as the shape of the contact area. Furthermore, [Giannakopoulos \(2006\)](#) presented analytical results of frictionless and adhesionless contact of flat, linear elastic and viscoelastic isotropic surfaces by pyramid indenters. His analysis considered the standard shapes

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