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Numerical modeling of guided wave interaction with non-axisymmetric cracks in elastic cylinders

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ABSTRACT

A three dimensional (3D) hybrid method combining the classical finite element (FE) method with the semi-analytical finite element (SAFE) technique is developed. This hybrid method is employed to study the interaction of guided waves with non-axisymmetric damages in cylinders. The near field surrounding the damage is analysed with the 3D FE method. The solution is expanded into sums of guided modes on both inlet and outlet cross-sections. Such eigenmode expansions enable separation into ingoing and outgoing waves, i.e., incident, reflected and transmitted waves. Using the SAFE method, elastic guided modes are then computed at the aforementioned cross-sections thus reducing the analysis to two dimensions (2D). The amplitudes of the incident modes are imposed, whereas those of the scattered modes are determined by solving the global system of the 3D hybrid FE-SAFE model. In this paper, a formula is proposed for the calculation of eigenforces and modal power flows from eigendisplacements and SAFE matrices. This has the advantage of simplifying the post-process of load eigenvectors in hybrid FE-SAFE methods. Results obtained for a vertical free-end cylinder are in good agreement with those published in the literature. Moreover, first results of the interaction of the fundamental compressional, flexural and torsional Pochhammer–Chree modes with non-axisymmetric vertical cracks are obtained and discussed. Then, the interactions of the fundamental compressional mode with oblique free-ends and cracks are briefly addressed. The power balance is shown to be satisfied with a good accuracy.

1. Introduction

Non-destructive testing (NDT) techniques using guided ultrasonic waves constitute viable inspection means for tubes, pipes and plates owing to their potential to carry out energy over long distances and their sensitivity to internal damages. Nevertheless, the interaction of such waves with damages is complex since elastic guided waves are multi-modal and dispersive. This highlights the essential role of modeling techniques in providing a better understanding of and a deeper insight into the nature of phenomena arising from scattering problems.

Various analytical approaches have been proposed in the literature. For instance, one can cite quasi-static approximations based on the *S*-parameter formalism for plates (Auld, 1973) and tubes (Ditri, 1994), and modal decomposition methods recently developed for plates (Le Clézio et al., 2002; Castaings et al., 2002; Demma et al., 2003; Shkerdin and Glorieux, 2004, 2005). Such methods are fast from a computational point of view but generally limited to simple geometries with horizontal or vertical discontinuities.

Fully numerical approaches based on transient finite element (FE) analyses have been conducted for plates (Alleyne and Cawley, 1992; Lowe and Diligent, 2002; Benmeddour et al., 2008a,b) and pipes (Lowe et al., 1998; Demma et al., 2004; Ma et al., 2006). The main advantage of these approaches is that complex-shaped waveguides or damages can be handled with standard FE codes. However, transient FE models are time consuming, which often limits their practical use to two-dimensional problems and short propagation distances. Besides, a modal post-processing step in the frequency domain must be performed, which requires an *a priori* knowledge of guided modes. Recently, Gunawan and Hirose (2004) have developed a mode-exciting method to analyse the interaction of Lamb waves with defects. The originality of their work is in the analysis of a finite plate model using a standard FE or boundary element (BE) method software with a time-harmonic regime. However, in addition to the *a priori* knowledge of guided modes, their analysis requires a great number of computations at high frequencies. Moreau et al. (2006) and Predoi et al. (2008) have used an existing FE software with absorbing regions at the edge of plates. Harmonic FE solutions have been post-processed with the help of a general orthogonality relation.

Hybrid approaches, combining a FE method and the so-called normal mode expansion technique have been developed for studying wave scattering by damages in plates (Karunasena et al., 1991,

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