International Journal of Solids and Structures 48 (2011) 2576–2584

Contents lists available at ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

Hierarchical structural optimization of laminated plates using polar representation

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ARTICLE INFO

Article history:
Received 22 October 2010
Received in revised form 12 May 2011
Available online 25 May 2011

Keywords:
Cross/angle ply laminates
Composite uncoupled laminates
Orthotropy
Elastic properties
Variable stiffness

ABSTRACT

A novel methodology for the design of optimal uncoupled laminated plates under membrane only or bending only loading is introduced. This approach is supported by the polar representation of anisotropy. First, topology optimization, aimed at maximizing global stiffness of the structure, allows to find an optimal distribution of polar components. Then, based on the latter structural results, a matching feasible lamination sequence is determined.

1. The problem of designing thin composite structures

Structural optimization is the process of designing structures that gives the best response to a given set of criteria. Topology optimization allows the definition of the optimal distribution of material, within a given domain and under given loads, with respect to a given criterion (i.e. maximizing global stiffness).

Applied to composite materials such as laminates, this methodology is twofold: it involves structural design – via topology optimization – as well as the definition of the local architecture of the material (i.e. fiber orientations and stacking sequence), which is likely to change from one point of the structure to another, leading to variable stiffness laminates.

In this paper, topology optimization is performed via a numerically efficient and convergent iterative algorithm introduced by Allaire and Kohn (1993). In order to minimize the compliance of the structure, each iteration is constituted by two optimization steps:

(i) local minimizations with fixed stress field,
(ii) global minimization with fixed material properties.

The major novelty of this work lies in the use of the polar representation of plane anisotropy in order to minimize complementary energy. This offers several benefits. First, in case of an orthotropic material, resolution of step [i] is performed through closed-form relations. Besides, feasibility of the optimal laminates is ensured by taking into account conditions on the orthotropy shape in the optimization process. Moreover, the optimization methodology combined with the polar method can be applied to both in-plane loading (Hammer et al., 1997; Hammer, 1999) and bending loading. Finally, the use of the polar parameters allows to solve independently the problem of designing the lamination of the optimized structure.

Indeed, topology optimization algorithm's output are fields containing the values of the optimal polar moduli in each element of the structure. We will show that it is then possible to find simple lamination sequences (such as angle-ply or cross-ply laminates) corresponding to these fields at every point of the structure.

2. The polar method

The polar method representation of a plane elasticity tensor is a representation with invariants, obtained by a transformation of coordinates, introduced by Verchery (1979), based upon a complex variables transformation: polar components have a physical significance linked to the elastic symmetries of material (Vannucci and Vincenti, 2007).

The link between the cartesian components of a 2D second order tensor such as the stress tensor σ and its polar components R, T and Φ is expressed by:

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0020-7683/$ - see front matter
doi:10.1016/j.ijsolstr.2011.05.015