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## Three-dimensional elastic displacements induced by a dislocation of polygonal shape in anisotropic elastic crystals

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## ABSTRACT

Dislocations and the elastic fields they induce in anisotropic elastic crystals are basic for understanding and modeling the mechanical properties of crystalline solids. Unlike previous solutions that provide the strain and/or stress fields induced by dislocation loops, in this paper, we develop, *for the first time*, an approach to solve the more fundamental problem—the anisotropic elastic dislocation displacement field. By applying the point-force Green's function for a three-dimensional anisotropic elastic material, the elastic displacement induced by a dislocation of polygonal shape is derived in terms of a simple line integral. It is shown that the singularities in the integrand of this integral are all removable. The proposed expression is applied to calculate the elastic displacements of dislocations of two different fundamental shapes, i.e. triangular and hexagonal. The results show that the displacement jump across the dislocation loop surface exactly equals the assigned Burgers vector, demonstrating that the proposed approach is accurate. The dislocation-induced displacement contours are also presented, which could be used as benchmarks for future numerical studies.

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## 1. Introduction

The presence of dislocations and induced elastic fields strongly influence the material properties of crystalline solids, such as the crystal growth (Rupert et al., 2009; Rajgarhia et al., 2009), the mechanical strength (Püschl, 2002; Kurzic, 2009) and other related physical properties (Bai et al., 2001; Gromov et al., 2010). The elastic fields induced by dislocations were widely studied in the past (Mura, 1963; Willis, 1970; Hirth and Lothe, 1982; Ting, 1996; Lubarda, 2003; Ghoniem and Han, 2005; Paynter and Nowell, 2005; Paynter and Hills, 2009). Generally, the elastic fields due to dislocation loops in three-dimensional (3D) homogeneous solids can be evaluated by the integrals of the point-force Green's function and its derivatives over the dislocation surfaces (Mura, 1963; Willis, 1970; Wang, 1996). These surface integrals for the stress field or strain field were reduced to the integrals along the dislocation line by means of Stokes' theorem (Mura, 1987). Moreover, Willis (1970) found the analytical expression of the stress field for a straight segment of a dislocation loop in an anisotropic solid. Gosling and Willis (1994) derived a line integral for the stresses due to an arbitrary dislocation in an isotropic half-space. Wang (1996) discussed the integral along curved segments and presented a derivation for the stress field induced by a curved dislocation loop. Obviously, reducing the surface integral to a line integral, or even to an analytical expression, is advantageous computationally and provides more accurate results and insight. However, in general, reductions to line integrals or analytical expressions have been applied for predictions of the dislocation-induced stress and/or strain field only, but not for the associated displacement field. As for the latter, Lerma et al. (2003) obtained the displacement field due to a symmetrical prism dislocation in isotropic solids. Ghoniem and Sun (1999) presented a line integral for an arbitrarily shaped dislocation loop in an isotropic material.

The major difficulty in reducing the surface integral in the expression for the displacement field induced by dislocations in anisotropic materials to a line integral is that the Stokes' theorem cannot be utilized. The displacement integral expression does not satisfy the conditions that are required in applying the Stokes' theorem. To the best of our knowledge, no explicit result or line integral expression has been reported for the elastic displacement field produced by the dislocation loops in 3D generally anisotropic media. In this paper, a line integral (from 0 to  $\pi$ ) expression for the elastic displacement field induced by a triangular dislocation is derived by using the point-force Green's function in the Stroh formalism. With this fundamental solution, the solution to an arbitrary polygonal dislocation can be constructed by the method of superposition. The influence of dislocation shape is included in the integrand.

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