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Predicting the deflection and sub-surface stress field within two-dimensional inhomogeneously elastic bonded layered solids under pressure

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ABSTRACT

This paper describes a Fourier series based solution method for the displacements and sub-surface stresses within a graded elastic layered solid under pressure. The solid is assumed to be in a state of plane strain and thus the derived solution is valid for two-dimensional problems. Whilst this method provides a fully analytic solution when the contact pressure is known exactly, it may also be used when the contact pressure is only known numerically (see Section 4). The solution given in this paper is generic and easily utilised to solve real problems as it requires only known physical characteristics of the solid under study and an applied surface pressure.

The solid consists of two distinct regions which are considered to be perfectly bonded. These comprise a graded elastic coating whose shear modulus varies exponentially with the depth coordinate and a homogeneously elastic substrate. As the stresses and displacements induced by the applied pressure decay very quickly outside of the contact region, the contact problem need only be solved in a small piece of the solid as the remainder is unaffected. It is found that accurate results are obtained when the contact problem is solved over a region of the solid 10 times larger than the contact region. This method as a result is computationally cheap to use as the number of Fourier modes needed to accurately capture the solution is small.

1. Introduction

Protective coatings are often used in engineering applications where motion and force is transmitted through direct contact. These coatings are usually deposited over a base material and may consist of one or more layers whose mechanical properties (e.g. hard or soft, thick or thin) are carefully tailored for the intended purpose and environment they must operate in. Finding the ideal coating requires a good understanding of the physical context of the application as well as the mechanical properties of both the coating and substrate.

Often, the mechanical properties of these coatings are far from homogeneous. It is common to see either a transition zone where the mechanical properties of the layer progressively morph into the ones of the substrate (e.g. Shulha et al., 2004) or coatings that are engineered in such a way that their properties continuously change throughout their thickness (Barbezat, 2008; Uozato et al., 2005). The latter materials are usually termed functionally graded (FGMs) and are a relatively new concept in material design (Suresh et al., 1998; Suresh et al., 1999).

Existing models that seek to approximate the contact problem involving FGMs tend to assume that the material is split into two distinct regions corresponding to the coating and substrate. The modulus of elasticity within the coating is assumed to depend on the depth coordinate in some pre-determined way whilst the substrate is assumed to be homogeneous. Common approximations let the modulus of elasticity within the coating follow either a simple power law (e.g. Giannakopoulos and Suresh, 1997) or an exponential variation (e.g. Guler and Erdogan, 2004).

Attempts to model the contact problem involving inhomogeneous materials have proved difficult as the fully three-dimensional problem tends to be resistant to analytical approaches. Giannakopoulos and Suresh (1997) presented some simple analytical models for the determination of the stresses and displacement within an inhomogeneous three-dimensional medium subject to a point force using both power law and exponential variations of the modulus of elasticity. The model solutions produced were then compared to numerical solutions obtained using the finite element method. A numerical method based on the Fast Fourier Transform (FFT) was proposed by Wang et al. (2010) to investigate partial slip contacts again in a three-dimensional context, however these authors give

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